Battle of the sexes and the prisoners dilemma philosophy essay

I’ve had two experiences in the cases of Battle of Sexes and Prisoner’s Dilemma. My friend Chris and I once had a dispute on which movie to watch – either “ Harry Potter” or “ Toy Story”. Both of us would like to watch both of them, but Chris would like to watch Harry Potter while I prefer Toy Story. Eventually, I suggested to watch Harry Potter first and Toy Story later.

The other case happened when I was a kid. I used to lie to my mum when I was young. I always failed to hand in my homework on time. However, my teacher reported to my mom about the poor quality of my work. So my mum once inspected me and caught me for watching cartoons before finishing my homework. Then, she subjects me to study sessions at school for a year so I could catch up with my school work. However, in this year, my mom was disappointed about my attitude and I could no longer enjoy watching cartoons.

I’ve realized I could analyze both scenarios with Game Theory, specifically Battle of Sexes and Prisoner’s Dilemma. And both two games belong to “ Two-Person Non-Zero Sum Game”, which describes a situation where a participant’s gain or loss is not balanced by the gains or losses of the other participant. Many common social dilemmas fall into this category, such as Centipede Game, Dictator Game (these will not be discussed in the essay) and etc.

Utility Theory

To support the claims of these games, the term utility has to be introduced. Utility refers to a measure of relative satisfaction. However, how much pain or pleasure a person feels and psychological effects can hardly be measured.

In order to create a measurable platform for mathematicians to examine the best probable solution, numbers are assigned to notate utility for the concrete numerical reward or probability a person would gain. For instance, if I watch cartoons in order to escape from 50 difficult math questions, I will gain 50 util. Although this is relatively subjective, it is better to set a more objective measurement than having pure language description.

Non-cooperative

In Game Theory, we will always deal with games that allow players to cooperate or not in advance. A cooperative game refers to a game in which players have complete freedom of communication to make joint binding agreements. On the other hand, a non-cooperative game does not allow players to communicate in advance. Rationally, players would make decisions that benefit them the most. However, in some cases, like the Battle of Sexes and Prisoner’s Dilemma, the common interests would not be maximized by their “ selfishness”.

Zero Sum Game

Zero-sum describes a situation in which a participant’s gain or loss is exactly balanced by the losses or gains of the other participant(s). If there are n participants and their outcomes are notated as O1, O2 … On. Mathematically speaking,

If player 1 uses a set of strategy A = (A1, … , Am) and player 2 uses B = (B1, … , Bn), the outcome Oij would have the probability xiyj, where both 1â‰¤ i, j â‰¤ m, n . The

M1(x, y) = —– player 1, and

M2(x, y) = —–player 2

Basically they are the expected value function for discrete X which express the expected value of their utilities. XiYj is the probability to certain decision while Ai and Bj are the respective decisions of player 1 and 2.

The motivation of player is 1 to maximize M1 and of player 2 to maximize M2. In a competitive zero-sum game we have zeros of the utility functions so that

M2(x, y) = -M1(x, y)

which led to the term zero-sum.

Therefore, it is never advantageous to inform your opponent the strategy you plan to adopt since there is only one clear winner and clear loser. So now we understand the concept that players cannot cooperate with each other. However, Battle of Sexes and Prisoner’s Dilemma could maximize the outcome through cooperation because they are non-zero sum game.

M2(x, y) â‰ -M1(x, y).

Notation

Suppose we have two players Chris (C) and Me (M) in a game which one simultaneous move is allowed for each player – the players do not know the decision made by each other. We will denote two sets of strategies as follows:

S1: C = {C1, C2, C3 ... Cm}

S2: B = {M1, M2, M3 … Mn}

A certain outcome Oij is resulted from a strategy from each player, Ai and Bj.

Matrix:

So if I pick strategy 1, Chris picks strategy 2 for himself, the outcome would become O21. Therefore, each sets of strategy between Chris and me would have a distinctive outcome, in which there are mn possibilities. However, in this essay we do not deal with many decisions, mostly 2 per person – Harry Potter (HP) or Toy Story (TS), or Honest or Dishonest. So it would come down to a 2×2 matrix, like the following diagram shown in Two-Person Non -Zero-Sum Game.

Two person Non-Zero Sum Game

Non-zero-sum games are opposite to zero-sum games, and are more complicated than the zero-sum games because the sum could be negative or positive. And a two person non-zero sum game is only played by two players. In a non-zero-sum game, a normal form must give both payoffs, since the loss is not incurred by the loser, but by some other party. To illustrate a few problems, we should consider the following payoff matrix.

Payoffs shows as (Player 1, Player 2)

Player 1

Strategy A

Strategy B

Player 2

Strategy X

(8, 9)

(6, 5)

Strategy Y

(5, 10)

(1, 0)

Apparently, if we sum up the payoffs of player 1, we would have 8+6+5+1 = 20. While Player 2 would have the payoffs of 9+5+10 = 19. This has clearly illustrated on of the properties of a non-zero sum game. Moreover, even if their payoffs are equal, one more requirement has to be met. The sum of all outcomes has to be 0. Since we only have positive integers here, we can conclude that the sum of all outcomes in this case is strictly > 0. So this is a typical example of two-person non-zero sum game.

Introduction to Pure and Mixed Strategies

Suppose a player has pure strategies S1, S2…Sk in a normal form game. The probability distribution function for all these strategies with their respective probabilities:

P =

p1, p2 …pk are nonnegative and = 1 because the sum of the probability of all strategies has to be 1. A pure strategy is achieved when only one is equal to 1 and all other pm are 0. Then P is a pure strategy and could be expressed as P = . However, a pure strategy is also used in a mixed strategy. The pure strategy is used in mixed strategy P if some is > 0.

So in a micro-scale, there are many strategies in the pure-strategy set S; and in macro-scale, these strategy-sets contribute to a bigger profile P. We define the payoffs to P as following:

where m, k â‰¥ 1

But if the strategy set S is not pure, the strategy profile P is considered strictly mixed; and if all the strategies are pure, the profile is completely mixed. And in the completely mixed profile, the set of pure strategies in the strategy profile P is called the support of P. For instance, in a classroom has a pure strategy for teacher to teach and for student to learn. Then these strategies, teaching and learning, are the support of the mixed strategy.

Payoffs are commonly expressed as So let i (s1,…, sn) be the payoff to player i for using the pure-strategy profile (P1,…, Pn) and if S is a pure strategy set for player i. Then the total payoffs would be the product of the probability of each strategy in the strategy set S (ps) and the payoffs of each strategy (. So if we sum up all the payoffs:

I (P) = , which is again similar to the expected mean payoff function we set up in the zero-sum game section.

However, a key condition here is that players' choices independent from each others', so the probability that the particular pure strategies can be simply notated as . Otherwise, probability of each strategy is expressed in terms of other ones.

Nash Equilibrium

The Nash equilibrium concept is important because we can accurately predict how people will play a game by assuming what strategies they choose by implementing a Nash equilibrium. Also, in evolutionary processes, we can model different set of successful strategies which dominate over unsuccessful ones; and stable stationary states are often Nash equilibria.

On the other hand, often do we see some Nash equilibria that seem implausible, for example, a chess player dominates the game over another. In fact they might be unstable equilibria, so we would not expect to see them in the real world in long run. Thus, the chess player understands that his strategy is too aggressive and careless, which leads to continuous losses. Eventually he will not adopt the same strategy and thus is put back to Nash equilibrium. When people appear to deviate from Nash equilibria, we can conclude that they do not understand the game, or putting to ourselves, we have misinterpreted the game they play or the payoffs we attribute to them. But in important cases, people simply do not play Nash equilibria which are better for all of us. I lied to my mom because of personal interests. The Nash equilibrium in the case between my mom and me would be both being honest.

Suppose the game of n players, with strategy sets si and payoff functions I (P) = , for i = 1…n, where P is the set of strategy profiles. Let S be the set of mixed strategies for player i.

where m, k â‰¥ 1

The fundamental Theorem of a mixed-strategy equilibrium develops the principles for finding Nash equilibria. Let P = (P1…Pn) be a mixed-strategy profile for an n-player game. For any player i, let P-i represent the mixed strategies used by all the players other than player i. The fundamental theorem of mixed-strategy Nash Equilibrium says that P is a Nash equilibrium if and only if, for any player i = 1 … n with pure-strategy set Si and if s, s' Si occur with positive probability in Pi, then the payoffs to s and s', when played against P-i are equal.

Battle of Sexes

We shall begin with my example:

At the cinema (C: Chris, M: Me)

M1

M2

C1

(2, 1)

(-1,-1)

C2

(-1,-1)

(1, 2)

*Choice 1: Harry Potter

*Choice 2: Toy Story

The game can be interpreted by a situation where Chris and I could not make the choice that satisfies both of them. Chris prefers Harry Potter while I prefer a movie. Consequently, if we choose our preferred activities, they would end up at (C1, M2) where the outcomes would only be (-1,-1) because both of us would like to watch the movie together.

Thus the Utility Function (U): Utility from the movie + Utility from being together.

Considering a rather impossible situation where both of us do not choose our preferred options (C2, M1). This dilemma has put one of us sacrifice our entertainment and join the other, like (C1, M1) or (C2, M2). Thus the total outcome could be up to 3 util instead of -2 in the other two situations. Therefore, I made a decision to give up watching Toy Story and join Chris watching Harry Potter.

Let be the probability of Chris watching Harry Potter and be the probability of me watching Toy Story. Because in a mixed-strategy equilibrium, the payoff to Harry Potter and Toy Story must be equal for Chris. Payoff for me is and

Chris' payoff is . Since , , which makes . On the other hand, has to be 1-2/3 = 1/3.

Thus, the probability for (C1, M1) or (C2, M2) = and that for (C2, M1) and (C1, M2) =

Because both go Harry Potter (2/3)(1/3) = 2/9 at the same time, and similarly for Toy Story, and otherwise they miss each other. Both players do better if they can cooperate (properties of non-zero sum game), because (2, 1) and (1, 2) are better than .

We get the same answer if we find the Nash equilibrium by finding the intersection of the players' best response functions. The payoffs are as follows:

To find the payoffs of Chris relative to my probability, which is similar to probability distribution function (p. d. f.). Here are the cases

Similarly for player B

Thus. Chris would have a lower tendency for a positive payoff since his payoff tends to decrease if 0 â‰¤ < 2/3. Therefore, as a friend, I did not want Chris to have a negative payoff, so I decided to watch Harry Potter with him.

Prisoner's Dilemma

Now it is the situation of where I lied to my mom. Here's the action between me and my mom. I could choose to be honest or lie to my mom while my mom, on the other hand, could only trust me or suspect me of being dishonest. The payoff matrix is as follow (Me: I, Mom: M):

I1

I2

M1

(2, 2)

(0, 3)

M2

(3, 0)

(-1, -1)

*Choice 1: Honest/Trust

*Choice 2: Dishonest/Suspect

This situation is a prisoner's dilemma because it sets up a few key conditions. If both my mom and I choose to be honest, I would do the homework but I will not be subject to homework session for a year, and my mom will not be upset about me. So it results in the best mutual benefits (2, 2). If I lie to her and she trusts me, I am happy from watching cartoon (3, 0). But if she suspects me and I am honest, I would feel like a prisoner being suspected. (0, 3). And eventually, if I am dishonest and she suspects me, we would end up in a bad relationship (-1,-1). Interestingly, I would prefer (I2, M1) because I have the greatest personal utility. But if I go for greatest mutual benefits, I would choose (I1, M1).

Utility Function for Me: (UI): C + H + S + R

C = Utility from watching cartoon

H = Utility from doing homework

S = Utility from homework session

R = Utility from relationship with mom

Now, to further discuss Prisoner’s dilemma for all cases, we had rather set up some variables.

I1

I2

M1

(1, 1)

(-y, 1+x)

M2

(1+x,-y)

(0, 0)

Now let be the probability of I play I1 and be that of M playing M1 and x, y > 0. And now we could set up the payoff functions easily with these notations.

Which could be simplified into

is maximized when = 0, and similarly for be maximized when = 0, regardless of what each other does. So in fact it is a mutually defect equilibrium because the best-response for each other is not the best response for both of us. Therefore, one of us should sacrifice for the others or both of us cooperate to work out the best solution.

In real life, people should choose to cooperate with trust. Assume that there is a psychic gain > 0 for I and > 0 for M when both of us cooperate, in addition to the tempting payoff 1+x. If we rewrite the payoffs with these assumptions and equations, we get

Which can further be simplified into

The first equation shows that if player I will then play I1 and if , then player M will play M1. Apparently, I would have done it because the total mutual payoffs of (I1, M1) – both my mom and I are honest and trustworthy, would be higher than that of (I2, M1) – where I lie to my mom who trust me. This would happen, for instance, I could get 10 candy bars and my mom can enjoy watching TV if both of us are honest. In fact, many corporates in the real world result in such way; therefore, sometimes, cooperation with others could be beneficial to ourselves.

Conclusion