

Oc curves or  
operating  
characteristic curves  
education essay



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OC Curves or Operating Characteristic Curves refer to a graph of attributes of a sampling plan considered during management of a project which depicts the percent of lots or batches which are expected to be acceptable under the specified sampling plan and for a specified process quality.

The specified sampling plan may be singular, sequential or iterative and may be using a particular size of a sample depending upon the demands of the project and could yield the results of acceptance or rejection based on a specified criteria.

### **OC Curve Uses**

It helps in the selection of sampling plans

It aids in the selection of plans that are effective in reducing risks.

It can help in keeping the high cost of inspection low.

### **OC Curve Calculation**

There are two ways of calculating the OC curves.

Binomial Distribution

Poisson Formula

### **Types of OC Curve**

Type A - Gives the probability of acceptance for an individual lot coming from finite production.

Type B - Gives the probability of acceptance for lots coming from a continuous process.

Type C - Gives the long-run percentage of product accepted during the sampling phase. [2]

### **Operating Characteristic (OC) Curves:**

A common supplementary plot to standard quality control charts is the so-called operating characteristic or OC curve (see example below). One question that comes to mind when using standard variable or attribute charts is how sensitive is the current quality control procedure? Put in more specific terms, how likely is it that you will not find a sample (e. g., mean in an X-bar chart) outside the control limits (i. e., accept the production process as “ in control”), when, in fact, it has shifted by a certain amount? This probability is usually referred to as the (beta) error probability, that is, the probability of erroneously accepting a process (mean, mean proportion, mean rate defectives, etc.) as being “ in control.” Note that operating characteristic curves pertain to the false-acceptance probability using the sample-outside-of- control-limits criterion only, and not the runs tests described earlier.

Operating characteristic curves are extremely useful for exploring the power of our quality control procedure. The actual decision concerning sample sizes should depend not only on the cost of implementing the plan (e. g., cost per item sampled), but also on the costs resulting from not detecting quality problems. The OC curve allows the engineer to estimate the probabilities of not detecting shifts of certain sizes in the production quality. [5]

Using OC Curves:

**Acceptance Sampling Plan:** These sampling plans consist of a sample size and a decision rule. The sample size is the number of items to sample or the number of measurements to take. The decision rule involves the acceptance limit(s) and a description of how to use the sample result to accept or reject the lot.

**Design Method:** To design a sampling plan by the two-point method, the designer specifies two points on the Operating Characteristic Curve (OC-Curve).

These two points define the acceptable and unacceptable quality levels for the purpose of acceptance sampling. The two points also determine the risks associated with the acceptance/rejection decision. You will find a more thorough discussion of the methodology in the Tutorial.

**Discrimination:** The sampling plans will require enough testing and inspection to discriminate between acceptable and unacceptable quality levels at the probability levels that the designer selects.

**Reduce Cost:** The sampling plans will save you time and effort by using no more testing and inspection than necessary in order to minimize cost.

**Articulation:** These sampling plans enable you to quickly know, describe, and explain the meaning of your acceptance rules — using operating characteristic curves to state the probabilities and risks.

**Accuracy:** We have refined the supporting software over many years of application by Quality and Reliability Engineers, Technicians, and

Consultants. In addition to sampling plan applications, it is very suited to classroom training.[1]

Operating Characteristic (OC) Curve:

The OC Curve is used in sampling inspection. It plots the probability of accepting a batch of items against the quality level of the batch.

The figure shows an 'OC' (Operating Characteristic) Curve for a sample of 50 items taken from a batch of 2000 and using a critical acceptance number 'c' of 2 (the batch will be accepted if there are two or less defectives in the sample). From the curve you can see that there is about a 23% probability of accepting a batch that contains 8% of defective items.

When designing a sampling plan it is usual to decide on two points, the AQL and LQL and the associated Producer's Risk and Consumer's Risk. The necessary sample size and acceptance number for the curve to pass through these points is then calculated and hence the shape of the curve.

OC Curves are mainly associated with sampling inspection but they are also used to find the Average Run Length in control charts.[3]

Using OC Curves in Design of Reliability Tests

Operating Characteristic (OC) curves are powerful tools in the field of quality control, as they display the discriminatory power of a sampling plan. In this article, we explain the applications of OC curves in reliability engineering.

In quality control, the OC curve plots the probability of accepting the lot on the Y-axis versus the lot fraction or percent defectives ( $p$ ) on the X-axis [1], as illustrated in Figure 1.

#### Figure 1: OC Curves in Quality Control

Based on the number of defectives in a sample, the quality engineer can decide to accept the lot, to reject the lot or even, for multiple or sequential sampling schemes, to take another sample and then repeat the decision process.

In reliability engineering, the OC curve shows the probability of acceptance (i. e. the probability of passing the test) versus a chosen test parameter. This parameter can be the true or designed-in mean life (MTTF) or the reliability ( $R$ ), as shown in Figure 2.

#### Figure 2: Probability of Acceptance Versus MTTF or Reliability

The probability of acceptance,  $P(A)$ , can be represented by the cumulative binomial distribution [2]:

which gives the probability that the number of failures observed during the test,  $f$ , is less than or equal to the acceptance number,  $c$ , which is the number of allowable failures in  $n$  trials. Each trial has a probability of succeeding of  $R$ , where  $R$  is the reliability of each unit under test (an analog to the probability of success in each Bernoulli trial). The reliability OC curve is developed by evaluating the above equation for various values of  $R$ .

In Table 1, we calculate the cumulative binomial probability for different levels of reliability and  $c = 2$  allowable failures in  $n = 10$  test samples.

Table 1: Probability of Acceptance for Various Values of Reliability when  $(n, c) = (10, 2)$

Reliability

P(A)

0.00

0.0000

0.05

0.0000

0.10

0.0000

0.15

0.0000

0.20

0.0001

0.25

0.0004

0.30

0.0016

0.35

0.0048

0.40

0.0123

0.45

0.0274

0.50

0.0547

0.55

0.0996

0.60

0.1673

0.65

0.2616

0.70



0.3828

0.75

0.5256

0.80

0.6778

0.85

0.8202

0.90

0.9298

0.95

0.9885

1.00

1.0000

A similar method can be used to develop an OC curve for a reliability demonstration test. We can use Weibull++'s Design of Reliability Tests (DRT) tool to simplify the calculation of the probability of acceptance values. The DRT can be accessed by either choosing Design of Reliability Tests from the Tools menu or clicking the DRT icon on the General toolbar.

On the Non-Parametric Binomial page of the DRT, we can calculate the confidence level for each value of reliability. The probability of acceptance used to construct the OC curve will equal  $1 - CL$ , where CL is the confidence level, since the binomial lower one-sided confidence limit is equal to the beta risk and equal to  $1 - CL$  [2, 3 and also see Type I and Type II Errors and Their Applications in the June, 2008 issue of the Reliability Hotwire].

For example, if reliability  $R = 90\%$ , number of units  $n = 10$ , and number of allowable failures  $c = 2$ , the confidence level is  $CL = 0.0702$ , as shown in the calculation in Figure 3. So, in this case,  $P(A) = 1 - CL = 1 - 0.0702 = 0.9298$ .

Figure 3: Weibulls++ Design of Reliability Tests Tool, Used to Calculate Confidence Level

By plotting the probability of acceptance versus reliability, we get the OC curve for this test, as shown in Figure 4.

Figure 4: OC Curve Corresponding to Table 1 Values when  $(n, c) = (10, 2)$

The selection of a reliability demonstration test plan depends upon the degree of risk that is acceptable and upon the cost of testing [4].

Comparison of different OC curves can provide an indication of relative risk.

In Figure 5 we examine the Type I (alpha or producers risk) and Type II (beta or consumers risk) errors for two different OC curves with a constant ratio of allowable defects to sample size  $c/n = 0.2$ .

Figure 5: Effect of Sample Size on Alpha and Beta Risks

For a low reliability, e. g.  $R = 0.72$ :

For  $(n, c) = (10, 2)$  the probability of acceptance  $P(A)$  is 0.43.

For  $(n, c) = (30, 6)$  the probability of acceptance  $P(A)$  is 0.22.

So in this region, by increasing the sample size we are lowering the beta risk (i. e. the risk of accepting a test when in fact the reliability is lower than the acceptable minimum).

For a higher reliability, we observe a similar behavior: an increased sample size reduces the alpha error of rejecting a test when in fact we have reliability higher than required.

The steeper the OC curve, the smaller the alpha and beta errors, in other words the greater the discriminatory power. If the alpha and beta errors were zero, the "ideal Reliability OC curve" would look like a step function, as shown in Figure 6. In practice, this can never be obtained, unless the whole population is tested and there are no errors in identifying failed versus passed units.

Figure 6: The Ideal OC Curve

The selection of the appropriate OC curve for a test depends on the relative risk factors and the associated costs that the organization is willing to afford at each phase of a project. For example, during product development, an OC curve with high alpha and beta risks might be selected, and during manufacturing, a more conservative curve with lower alpha and beta risks might be chosen.

We can design OC curves for specified levels of alpha and beta risk. Again assuming binomial sampling, we can derive the acceptance number,  $c$ , and sample size,  $n$ , by specifying the required alpha and beta values in the system of non-linear equations below:

(1)

(2)

For example, let's say that a manufacturer wants to have an alpha risk,  $\hat{\alpha}$ , of equal to or lower than 0.1, when the true value of reliability is equal to or higher than  $R_1 = 95\%$ . In other words, the manufacturer wants the probability of rejecting a lot with a true reliability above 95% to be up to 10%. By using Eqn. (1), we obtain a family of curves with different combinations of sample size and acceptance number. One pair of values of sample size,  $n$ , and acceptance number,  $c$ , that satisfies equation (1) is  $(n, c) = (94, 7)$ . Another combination is  $(n, c) = (22, 2)$ . The plot of these OC curves is shown in Figure 7.

Figure 7: OC Curves for Specified Alpha Risk

In a similar way, we can design an OC curve for a specific level of beta risk, based on Eqn. (2). Let's say a customer wants to have a beta risk of equal to or lower than 0.15, when the reliability  $R_2$  is equal to or lower than 80%. In other words, the customer wants the probability of accepting a lot with true reliability lower than 80%, to be up to 15%. By using equation (2), we can solve for the pair of values  $(n, c) = (46, 8)$ . Another combination is  $(n, c) = (22, 2)$ . Figure 8 shows the plot of these OC curves.

**Figure 8: OC Curves for Specified Beta Risk**

If we solve equations (1) and (2) simultaneously, we can define OC curves that have specific alpha and beta risks for certain levels of reliability. For example the OC curve  $(n, c) = (22, 2)$  in the examples above reflects an alpha risk of 0.1 for  $R_1 = 0.95$  and at the same time a beta risk of 0.15 for  $R_2 = 0.80$ .