

# Math problems



Find the least integer  $n$  for which  $p_n(2)$  approximates  $f(2)$  with three decimal place accuracy From  $f(a+h)$  approximately  $f(a)+h f'(a)$

When  $h$  is small enough in terms of value of  $f'(a)$  and  $f(a)$

it is possible to approximate the value of

$f(a+h)$

For this case

let approximate the value

Of 2.1

Therefore 2.1 can be expressed as

$2.1 = 2 + h$  where

$h = 0.1$

Assuming  $f(x) = x$

Then  $f'(x) = 1/2x$

Therefore, by linear approximation formula

$x+h = x + h/2x$

And then

$2.1 = 2 + 0.1/2 =$

$2.1 = 2 + 0.035$

$= 1.4495$

Use Taylor polynomials to estimate the following to within 0.01

$e^{0.8}$

$e^x = 1 + x + x^2/2 + x^3/6 + \dots + x^n/n!$

$2/3^n$

$e^{0.8} = 1 + 0.8 + 0.8^2/2 + 0.8^3/6 + \dots + 0.8^n/n!$

$2/3^n$

$= 1 + 0.8 + 0.8^2/2 + 0.8^3/6$

<https://assignbuster.com/math-problems/>

$$2^3$$

$$= 1 + 0.8 + 0.64 + 0.212$$

$$2^6$$

$$= 1 + 0.8 + 0.32 + 0.0353$$

$$= 2.1553$$

Expand as indicated

$$\ln(x^2)$$

Let  $x^2$  be  $(x-1)^2$

Where 2 is constant

= then

$$\ln(x-1)^2 = 2\left\{\frac{(x-1)^2}{(x+1)^2}\right\} + \frac{1}{3}\left\{\frac{(x-1)^3}{(x+2)^3}\right\} + \frac{1}{5}\left\{\frac{(x-5)^5}{(x+5)^5}\right\}$$

For  $x > 0$

For

$$(a+b)^n = a^n + \frac{n}{1!} a^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2.$$

For this case, let 1 be a and  $2x$  be b

Therefore,

$$(1-2x)^{-3}$$

$$= 1^{-3} + \frac{3}{1!} 1^{-3} 2x + \frac{-3(-4)}{2!} 1^{-5} (2x)^2 + \dots$$

$$= -1 + \left\{ \frac{-3}{1!} 1^{-4} (2x) \right\} +$$

$$\left\{ \frac{-3(-4)}{2!} 1^{-5} (2x)^2 \right\} + \dots$$

$$= -1 + 6x + 24x^2 + \dots$$

$$= 24x^2 + 6x - 1 + \dots$$

Find interval of convergence

$$(-1)^k \left(\frac{2}{3}\right)^k (x+1)^k$$

$$\lim_{k \rightarrow \infty} \frac{(-1)^{k+1} \left(\frac{2}{3}\right)^{k+1} (x+1)^{k+1}}{(-1)^k \left(\frac{2}{3}\right)^k (x+1)^k}$$

$$(-1)^k \left(\frac{2}{3}\right)^k (x+1)^k$$

$$\lim_{x \rightarrow 1} (-1)^{2/3} (x+1)$$

$$1$$

$$= -2/3 (x+1)$$

$$= -(x+1) \lim_{x \rightarrow 1} 2/3$$

$$= -x-1 \lim_{x \rightarrow 1} 2/3$$

$$= -2/3 x+1 \text{ Therefore interval}$$

$$-2/3 x+1 < 1 \text{ Convergence.}$$

$$2 \ 1/k \ k$$

$$(x-2) \ k$$

$$K \ (k+1) \ (k+2)$$

$$\lim_{k \rightarrow \infty} 2 \ 1/r \ k$$

$$(x-2) \ k \ 1/k$$

$$K \ (k+1) \ (k+2) \ (k+1)$$

$$\lim_{k \rightarrow \infty} 2 \ 1/k \ k \ (x-2) \ k+1$$

$$0$$

$$k \ (k+1)^2 \ (k+2)$$

$$= x-2 \ 2 \ 1/k \ k \ (x-2) \ k+1$$

$$k \ (k+1)^2 \ (k+2)$$

$$= 0$$

$$\text{Therefore, } f = 0 < 1$$

Evaluation of the given limits

$$\lim_{x \rightarrow 1} e^x - 1 - x$$

$$x \tan^{-1} c$$

Using hospital rule,

$\lim_{x \rightarrow 0} (e^x - 1) - x$

$x \tan^{-1} c = \lim_{x \rightarrow 0} (e^x - 1)$

$\tan^{-1} c$

As the  $e^x - 1$  and  $\tan^{-1} c$  tends to zero, then

$\lim_{x \rightarrow 0} (e^x - 1)$

$\tan^{-1} c$

$= 1 = +$

0

Estimate within 0.01

1

$e^{-3x} dx$

0

$= [e^{-3x}]_1^0$

0

$= [e^{-1} - e^0]$

$= [0.368 - 1]$

$= -0.632$

Reference

Karner, G and Kuich, W, (1997). "Characterizations of Abstract Families of Algebraic Power Series".