Capital market line

The capital Asset pricing Model

In 1959 Markowitz did the groundwork for the capital asset pricing theory (CAPM). He introduced the notion of mean-variance efficient portfolio. According to him it is optimal for an investor to hold a mean-variance efficient portfolio. The mean-variance efficient portfolio is a portfolio for an investor where he minimizes the portfolio return, given the expected return and maximizes expected return, given the variance. Later Sharpe (1964) and Lintner (1965b) further developed the work of Markowitz. In their work it has been showed that if the investors' expectations are homogeneous and when the hold the mean-variance efficient portfolio then in the nonexistence of market friction the market portfolio will be a mean-variance efficient portfolio.

There are two basic building blocks to derive the CAPM: one is the capital market line (CML) and the other one is the security market line (SML). In CAPM the securities are priced in a way where the expected risks are compensated by the expected returns. As we will be investigating different form of CAPM in this work it is worthy to review the basic notions of CML and SML.

Capital Market Line

The capital market line (CML) conveys the return of an investor for his portfolio. As we have already mentioned, there is a linear relationship exists between the risk and return on the efficient portfolio that can be written as follows:

Here we can consider the expected return on a portfolio as a sum of the returns for deferring consumption and a premium for bearing the risk underlying the portfolio. It is noteworthy here that the CML is compelling only for the efficient portfolio.

On the Other hand the SML specifies the return what an individual expects in terms of a risk-free rate and the relative risk of a portfolio.

The Beta is interpreted as the amount of non-diversifiable risk intrinsic in the security relative to the risk of the efficient market portfolio.

The utility function of the market agent is either quadratic or normal

All the diversifiable risks are eliminated

The efficient market portfolio and the risk-free assets dominate the opportunity set of the risky asset.

We can use the security market line can be used to test whether the securities are fairly priced.

Single-factor CAPM

In practice, to check the validity of the CAPM we test the SML. Although CAPM is a single period ex-ante model, we rely on the realised returns. The reason being the ex ante returns are unobservable. So, the question which becomes so obvious to ask is: does the past security return conform to the theoretical CAPM?

We need to estimate the security characteristic line (SCL) in order to investigate the beta. Here the SCL considers the excess return on a specific

security j to the excess return on some efficient market index at time t. The SCL can be written as follows:

Many early studies (e. g. Lintner, 1965; Douglas, 1969) on CAPM focused on individual security returns. The empirical results are off-putting. Miler and Scholes (1972) found some statistical setback faced when using individual securities in analyzing the validity of the CAPM. Although, some of the studies have overcome the problems by using portfolio returns. In the study by Black, Jensen and Scholes (1972) on New York stock exchange data, portfolios had been formed and reported a linear relationship between the beta and average excess portfolio return. The intercept approaches to be negative (Positive) for the beta greater than one (less than one). Thus a zero beta version was developed of the CAPM model. The model was developed in a model where the intercept term is allowed to take different values in different period. Fama and Mcbeth (1973) extended the work of Black et al (1972). They showed the evidence of a larger intercept than the risk neutral rate. They also found that a linear relationship exists between the average returns and the beta. It has also been observed that this linear relation becomes stronger when we work with a dataset for a long period. However, other subsequent studies provide weak empirical evidence of this zero beta version.

Multifactor Models

So far we have not talked anything about the cross sectional variation. In many studies we have found that market data alone cannot explain the cross sectional variation in average security returns. In the analysis of CAPM some variables like, ratio of book-to-market value, price-earning ratio,

macroeconomic variables, etc are treated as the fundamental variables. The presence of these variables account for the cross-sectional variation in expected returns.

Fana and French (1995), in their study showed that the difference between the return of small stock and big stock portfolio (SMB) and the difference between high and low book-to-market stock portfolio (HML) become useful factor in cross sectional analysis of the equity returns. Chung, Johnson and Schill (2001) found that the SMB and HML become statistically insignificant if higher order co-moments are included in the cross sectional portfolio return analysis. We can infer from here that the SMB and HML can be considered as good proxies for the higher order co-moments. Ferson and Harvey (1999) made a point that many econometric model specifications are rejected because they have the tendency of ignoring conditioning information.

On the other hand arbitrage pricing theory (APT) by Ross (1976) shows that we do not need the condition of man-variance optimization for all the vendors. APT outperforms CAPM

CAPM with higher-order co-moments

We know that the unconditional security return distribution is not normal. Moreover, the mean and variance of security returns are not sufficient enough to characterise the distribution completely. Thus it encourages the researchers to look for the higher order co-moments. In practice we estimate the skewness (third moment) and kurtosis (fourth moment). In many studies researchers paid attention to the validity of CAPM in the presence of the higher order co-moments and their effects on the asset pricing. In many

studies skewness has been incorporated in the asset pricing models and it provided mixed results.

Some studies incorporated conditional skewness in their models. For example Harvey and Siddique (2000) investigated an extended version of CAPM. Since the conditional skewness confines the asymmetry in risk, this version of CAPM is usually preferred over the fundamental one. In recent times, this concept of conditional skewness has become very useful in measuring the value at risk. From the study of Harvey and Siddique (2000) we notice that the conditional skewness captures the variation in cross-sectional regression analysis of expected returns significantly. This also holds true when factors based on size and book-to-market are also considered.

In some studies we see that in determining the security valuations, the non-diversified skewness and kurtosis play an important role. Fang and Lai (1997) reported a four-moment CAPM and in their study they showed that systematic variance, systematic skewness and kurtosis contribute to the risk premium of the underlying asset.

Conditional asset pricing models

Levy (1974) suggested to estimate different betas for bull and bear markets. Following that suggestion, Fabozzi and Francis (1977) estimated the betas for bull and bear markets. However, they didn’t find any evidence for beta instability. However, in another work Fabozzi and Francis (1978) reported that investors need a positive premium in order to accept the downside risk. On the other hand a negative premium corresponds with the up market beta.

This up market beta is considered as a more appropriate measure of portfolio risk.

There are few other studies examined the randomness for beta. Kim and Zumwalt (1979) examined the variation in returns on portfolios in both up and down markets. They concluded that the up market comprises the months for which the market returns exceed the average market return, the average risk neutral rate and zero. They specified three measures to identify what make up an up and down market. Those months for which the market return exceeds that the average market return and when it is above the risk free rate or greater than zero constitute the up market. They observed that the respective betas of the down market is more accurate measure for the portfolio risk than the single beta we see in the conventional CAPM. In an investigation on risk-return relationship Chen (1982) allowed the beta to be non-stationary and observed that investor need compensation when they assume downside risk no matter whether the betas are constant or changing. The concluded the same about the down market risk that Kim (1979) did. Bhardwaj and Brooks (1993) concluded that the systematic risks are different in bull and bear time periods. The also classified the market as Kim and Zumwalt (1979) did but instead of comparing the market return with mean return they compared it with the median return.

Pettengill, Sundaram and Mathur (1995) observe that if we use the realized return then the beta-expected return relationship becomes conditional on the excess market return. From that study we that there exists a positive relationship between beta and ecpected return during a up market. In line with their study, Crombez and Vennet (2000) studied the conditional

relationship between asset return and beta. They concluded that beta is a dependable meter in both bull (upward market) and bear market (downside risk). For different kind of specifications of the up and the down market this beta factor becomes robust and the investors can increase the expected asset return by considering the up and down market separately. Therefore different moments vary and correspond to the up and the down market. Galagedera and Silvapulle (2002) analyzed the asset return and the higher order co-moments in both bull and bear market and suggested that in the skewed market return distribution, the excess return is related to the systematic co-skewness.

Early Empirical Test

There are three relationships between expected return and market beta which is implied by the model. First, the expected returns on all the underlying assets are linearly related to their respective betas. Second, the premium for beta is positive which implies that the expected return on the market portfolio exceeds the expected return on assets. Moreover, the returns of these assets are uncorrelated with the expected return of market portfolio. Third, in the Sharpe-Lintner model we see that the underlying assets which are uncorrelated with the market portfolio have the expected returns which are equal to the risk neutral interest rate. In that model, if we subtract the risk free rate from the expected market return, we get the beta premium. Conventionally the tests of CAPM are based on those three implications mentioned above.

Test on Risk Premiums

Most of the previous cross-section regression tests primarily focus on the Sharpe-Lintner model's findings about the concept and the slope term which studies the relationship between expected return and the market beta. In that model they regressed the mean asset returns on the estimated asset betas. The model suggests that the constant term in the cross-section regression stands for the risk free interest rate. Eqn-page8_chicago and the slope term stands for the difference between market interest rate and risk free interest rate.

There are some demerits of the study. First of all, the estimated betas for individual assets are imprecise which creates the measurement error when we use them to explain average returns. Secondly, the error term in the regression has some common sources of variation which produces positive correlation among the residuals. Thus the regression has the downward bias in the usual OLS estimate. Blume (1970) and Black, Scholes and Jensen (1972) worked on overcoming the shortcomings of Sharpe-Lintner model. Instead of working on the individual securities they worked on the portfolios. They combined the expected returns and market beta in a same way that if the CAPM can explain the security return, it can also explain portfolio return. As the econometric theory suggests, the estimated beta for diversified portfolios are more accurate than the estimated beta for the individual security. Therefore, if we use the market portfolio in the regression of average return on betas, it lessens the critical problem. However, grouping shrinks the range of estimated betas and shrinks the statistical power as well. To tackle this researchers sort securities to create two portfolios. The

first one contains securities with the lowest beta and it moves up to the highest beta.

We know that when there exists a correlation among the residuals of the regression model, we cannot draw accurate inference from that. Fama and Macbeth (1973) suggested a method to address this inference problem. They ran the regression of returns on beta based on the monthly data rather than estimating a single cross-section regression of the average returns on beta. In this approach the standard error of the means and the time series means can be used to check whether the average premium for beta is positive and whether the return on the asset is equal to the average risk free interest rate.

Jensen (1968) noted that Sharpe-Lintner model also implies a time series regression test. According to Sharpe-Lintner model, the average realized CAPM risk premium see page10 explains the average value of an asset’s excess return. The intercept term in the regression entails that “ Jensen’s alpha”

In early studies we reject Sharpe-Lintner model for CAPM. Although there exists a positive relation between average return and beta, it’s too flat. In Sharpe-Lintner model the intercept stands for the risk free rate and the slope term indicates the expected market return in access of the risk neutral rate. In that regression model the intercept is greater than the risk neutral rate and the coefficient on beta is less than E(R_M)-R_f

In past several studies it has been confirmed that the relationship in between average return and beta is too flat (Blume: 1970 and Stambaugh: 1982).

With the low betas the constant term in the time series regression of excess asset return on excess market return are positive and it becomes negative for the high betas of the underlying assets.

In the Sharpe-Linter model, it has been predicted that portfolios are plotted along a straight line where the intercept equals the risk free rate, R_f, and the slope equals to the expected excess return on the market rate E(R_M)-R_f.

See page 12, write the results and restrictions

Testing whether market betas explain market betas

Both the Sharpe-Lintner and Black model predict that market portfolio is mean-variance efficient. The mean-variance efficiency implies that the difference in market beta explains the difference in expected return of the securities and portfolios. This prediction plays a very important role in testing the validity of the CAPM.

In the study by Fama and Macbeth (1973), we can add pre-determined explanatory variables to the month wise cross section regressions of asset return on the market beta. Provided that all the differences in expected return are explained by the betas, the coefficients of any additional variable should not be dependably different from zero. So, in the cross-section analysis the important thing is to carefully choose the additional variable. In this regard we can take the example of the study by Fama and MacBeth (1973). In that work the additional variables are squared betas. These variables have no impact in explaining the average asset return.

By using the time series regression we can also test the hypothesis that market betas completely explain expected asset return. As we have already mentioned that in the time series regression analysis, the constant term is the difference between the asset's average return and the excess return predicted by the Sharpe-Lintner model. We cannot group assets in portfolios where the constant term is dependably different from zero and this applies only the model holds true. For example, for a portfolio, the constant term for a high earning to price ratio and low earning to price ratio should be zero. Therefore, in order to test the hypothesis that betas suffice to explain expected returns, we can estimate the time-series regression for the portfolios and then test the joint hypothesis for the intercepts against zero. In this kind of approach we have to choose the form of the portfolio in a way which will depict any limitation of the CAPM prediction.

In past literatures, researchers tend to follow different kinds of tests to see whether the constant term in the time-series regression is zero. However, it is very debatable to conclude about the best small sample properties of the test. Gibbons, Shanken and Ross (1989) came up with an F-test for the constant term that has the exact-small sample properties and which is asymptotically efficient as well.

Tangency portfolio

For the tangency portfolio, this F-test builds a entrant by combining the market proxy and the average value of an asset's excess return. Then we can test if the efficient set and the risk free asset is superior to that one obtained by combining the market proxy and risk free asset alone. From the study of Gibbons, Ross, and Shanken (1989) we can also test whether

market betas are sufficient enough to explain the expected returns. The statistical test what is conventionally done is if the explanatory variables can identify the returns which are not explained by the market betas. We can use the market proxy and the left hand side of the regression we can construct a test to see if the market proxy lies on the minimum variance frontier.

All these early tests really do not test the CAPM. These tests actually tested if market proxy is efficient which can be constructed from it and the left hand side of the time series regression used in the statistical test. Its noteworthy here that the left hand side of the time series regression does not include all marketable assets and it is really very difficult to get the market portfolio data (Roll, 1977). So, many researchers concluded that the prospect of testing the validity of CAPM is not very encouraging.

From the early literatures, we can conclude that the market betas are sufficient enough to explain expected returns which we see from the Black version of CAPM. That model also predicts that the respective risk premium for beta is positive also holds true. But at the same time the prediction made by Sharpe and Lintner that the risk premium beta is derived from subtracting the risk free interest rate from the expected return is rejected. The attractive part of the black model is, it is easily tractable and very appealing for empirical testing.

Recent Tests

Recent investigations started in the late 1970s have also challenged the success of the Black version of the CAPM. In recent empirical literatures we see that there are other sources are variation in expected returns which do

not have any significant impact on the market betas. In this regard Basu's (1977) work is very significant. He shows that if we sort the stocks according to earning-price ratios, then the future returns on high earning-price ratios are significantly higher than the return in CAPM. Instead of sorting the stocks by E/P, if we sort it by market capitalization then the mean returns on small stocks are higher than the one in CAPM (Banz, 1981) and if we do the same by book-to-market equity ratios then the set of stocks with higher ratio gives higher average return (Statman and Rosenberg, 1980).

The ratios have been used in the above mentioned literatures associate the stock prices which involves the information about expected returns which are not captured by the market betas. The price of the stock does not solely depends on the cash flows, rather it depends on the present discounted value of the cash flow. So, the different kind of ratios discussed above play a crucial role in analyzing the CAPM. In line with this Fama and French (1992) empirically analyzed the failure of the CAPM and concluded that the above mentioned ratios have impact on stock return which is provided by the betas. In a time series regression analysis they concluded the same thing.

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Irrational Pricing of risk:

There are two groups who conclude that the empirical validity of CAPM is poor. The one of them is the behavioralists and the other group believes that we need more tedious and complicated asset pricing model. The first group argues that sorting the firms according to the different kind of ratios expose them to the investors and provides them the opportunity to overreact. When

the investors correct themselves from overreacting, that results high pay off for the value stocks and the pay off is low for the growth stocks. The second group argues that the assumptions of the CAPM theory are not realistic. For instance, the investors care about the distribution of mean and variance is really unrealistic. Investors care about other investment opportunity, their labour income and the dimensions of the risk. Therefore, the market betas cannot completely capture of an asset’s risk. Consequently differences in expected return cannot be completely captured by the difference in beta.

Metron (1973) extended the CAPM which is known as the intertemporal capital asset pricing model (ICAPM). This model is based on different assumptions. In the conventional CAPM, the investors only care about the pay off at the end of the period but in ICAPM, investors also care about the consumptions or the amount of asset they will have to invest in the next period.

In ICAPM, the investors prefer high expected return and low variance in return as they do in the CAPM. But along with that the investors also care about the covariance of the asset returns and state variables. Therefore, the combination of the portfolio which is optimal are multifactor efficient which implies that which has largest expected return.

ICAPM actually generalizes the rationale behind the CAPM. The portfolio becomes multifactor efficient when the short selling of the risky assets are allowed. This implies that there is a relationship holds between expected market return and beta risks. Ross’s (1976) arbitrage pricing theory (APT)

specified some of these state variables. Ideally ICAPM also specifies these state variables.