

# Arrow impossibility theorem essay sample



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Arrow's impossibility theorem represents a fascinating problem in the philosophy of economics, widely discussed for insinuating doubt on commonly accepted beliefs towards collective decision making procedures. This essay will introduce its fundamental assumptions, explain its meaning, explore some of the solutions available to escape its predictions and finally discuss its implications for political voting and elections. I will begin by giving some definitions and presenting the fundamental issue of social choice theory, consisting of the identification of an "ideal" device for preference aggregation, capable of converting individual rankings into collective ones, reflecting each individual's preference into an optimal societal choice.

Given a finite set of voters having to choose between a finite set of candidates, we call a voting system the function taking as input the voting preferences of each voter and returning as output a collectively valid ranking of the candidates. Majority voting is the voting system requiring that given two alternative options X and Y, X is preferred to Y by the group if the number of group members preferring X to Y exceeds that of members preferring Y to X. When group preferences are rational and transitive, for every pairwise comparison between two options, the group-wide valid outcome obtained applying majority voting is a unique winning alternative, which is said to be the "Condorcet winner". Sometimes though, group preferences are not rational and for each pairwise comparison a different winner emerges. In such case there is said to be a cycling majority and the situation represents a "Condorcet paradox".

Named after the distinguished economist and nobel laureate Kenneth Arrow, the "Arrow Impossibility Theorem" was first proposed and demonstrated in

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his book “ Social Choice and Individual Values”, published in 1951. The theorem states the impossibility of achieving an aggregation device capable of translating the ranked preferences of individuals into coherent societal rankings while meeting a set of minimal conditions necessary for it to represent a fair collective decision-making mechanism.

Such conditions are:

(U) Unrestricted scope: An acceptable voting system should process any coherent set of individual preferences ranking any number of alternatives. Preferences must be rationally well behaved, therefore at least complete and transitive.

(P) Pareto optimality: The voting system should respect unanimity: If every individual in the group prefers X to Y, the system must then rank X above Y in the collective ranking.

(I) Independence from irrelevant alternatives: The collective preference between X and Y should depend only on the single individuals preferences between those alternatives. The introduction of irrelevant alternatives into individual rankings should not affect the social one.

(N) Non-dictatorship: The voting system should account for the wishes of multiple voters: No single individual preferences should dictate in the group preferences regardless of those of all other individuals.

Even if these fundamental conditions appear so diverse and mutually unrelated that it’s difficult to see how they might conflict, taken together in the theorem they prove to be incompatible. This has been defined as the “  
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Arrow's paradox". The theorem's fundamental implications are the necessity of sacrificing at least one of the above assumptions in order to obtain a rational voting system and that social choice is subject to a trade-off between rationality and concentration of power. The existence of such trade-off poses serious obstacles for the identification of an optimal voting system in real society elections and has strong consequences in welfare economics and justice theory.

There are possible solutions to escape the paradox by accepting to relax one or more of the theorem fundamental assumptions. For example, by restricting the voting choice between just two alternatives, a coherent group decisions can always be obtained through majority voting. In politics this is commonly achieved using different methods among which are the assignment of agenda power to an individual, the elimination of defeated alternatives from the vote and the limitation of voting rounds. Individuals having the power of making such choices can actively influence the results of the elections and have their most preferred outcome implemented. Another popular approach consists of restricting the application of the voting system to a specific type of individual preferences denoted as "single-peaked", characterized by the presence of a most preferred alternative for each individual. Furthermore, in the context of a uni-dimensional policy space we can also apply "Black's median voter theorem" to identify the Condorcet winner in the societal ranking, consisting of the most preferred alternative of the median voter.

Despite the existence of a number of cases where it's possible to circumnavigate the paradox, many political problems present complex and

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multi-dimensional settings for which the limitations of voting systems highlighted by Arrow's theorem represent a serious obstacle and a currently unsolved issue. In conclusion, in his work Arrow proposes a set of reasonable conditions for defining a voting rule capable of generating consistent social choices within a general welfare context. Although, his findings demonstrate that any rule admitting dictatorship requires it while any other that requires nondictatorship contradicts at least one of the other conditions. In other words, the only decision-making method that is not flawed is a dictatorship.

This is frequently referred to as the “ social choice paradox”, a thesis that seems to undermine democracy under all effects by proving its most basic conditions impossible to coexist. Arrow encourages though, to accept the paradox as a challenge rather than a barrier to democracy, moving the focus of election theory from election results to the methods used to achieve them, pushing societies towards the development of ever-increasingly refined instruments for collective choice. Even if there's no “ ideal” election system, capable of ensuring the best collective decision for all members of society, certainly some existing ones are strongly preferable to others for the achievement of specific objectives and finally, in any given setting of collective choice, it is possible to question whether the reasonable criteria are truly reasonably applied within that particular context.

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