Bond pricing basic concepts

Economics, Financial Markets

P&L Explain – Bonds and Swaps Tony Morris antony.com MICS – DKS Manila Contents 1. Bond Pricing – basic concepts 2. P&L sensitivities of a bond i. PV01 ii. CS01 iii. Theta iv. Carry 3. Extension to interest rate swaps 1. Bond Pricing – basic concepts Let's say you have a 4 year 10% annual coupon bond, with a yield (' yield to maturity' or ' yield to redemption') of 12%. From this information, the price can be calculated as 93. 93%. The price is calculated by pricing each of the bond's cash flows using the yield to maturity (YTM) as a discount rate.

Why? Because the YTM is defined as the rate which, if used to discount the bond's cash flows, gives its price. We could picture it like this: Bond Cash Flows on a Time Scale Each fixed coupon of 10% is discounted back to today by the yield to maturity of 12%: 93. 93% = 10 + 10 + 10 + 110 (1. 12)1 (1. 12)2 (1. 12)3 (1. 12)4 All we are doing is observing the yield in the market and solving for the price. Alternatively, we could work out the yield if we have the price from the market.

Bond price calculators work by iteratively solving for the yield to maturity. For a bond trading at par, the yield to maturity and coupon will be the same, e. g. a four year bond with a fixed coupon of 10% and a yield of 10% would be trading at 100%. Note that bond prices go down as yields go up and bond prices go up as yields go down. This inverse relationship between bond prices and yields is fairly intuitive. For our par bond above, if four year market yields fall to 9% investors will be willing to pay more than par to buy the above market coupons of 10%. This will force its price up until it, too, yields 9%.

If yields rise to, say, 11% investors will only be willing to pay less than par for the bond because its coupon is below the market. For a detailed example of the bond pricing process, see Appendix 3. For now, note that the dirty price of a bond is the sum of the present values of the cash flows in the bond. The price quoted in the market, the so-called “ clean” price or market price, is in fact not the present value of anything. It is only an accountants’ convention. The market price, or clean price, is the present value less accrued interest according to the market convention. . P&L sensitivities of a bond As we saw above, the price of a bond can be determined if we know its cash flows and the discount rate (i. e. YTM) at which to present value them. The yield curve from which are derived the discount factors for a bond can itself be considered as the sum of two curves: 1. the “ underlying” yield curve (normally Libor), and 2. the “ credit” curve i. e. the spread over the underlying curve The sensitivity of the bond price to a change in these two curves is called: i. PV01, and ii. CS01 respectively.

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In terms of the example above, the discount rate of 12% might be broken down into, say, a Libor rate of 7% together with a credit spread of 5%. (Note, in the following, it is important not to confuse the discount rate, which is an annualised yield, and the discount factor, which is the result of compounding the discount rate over the maturity in question.) In addition to the sensitivities described above, we can also consider the impact on the price of the bond of a one day reduction in maturity. Such a reduction affects the price for two reasons:) assuming the yield curve isn’t flat, the discount rates will alter because, in general, the discount rate for time “ t” is not the same as that for time “ t-1” b) since one day has elapsed, whatever the discount rate, we will compound it based on a time interval that is shorter by one day The names given to these two sensitivities are, respectively:

Theta, and Carry Note that, of these four sensitivities, only the first two, i. e. PV01 and CS01, are “ market sensitivities” in the sense that they correspond to sensitivities to changes in market parameters.

Theta and Carry are independent of any change in the market and reflect different aspects of the sensitivity to the passage of time. i)PV01 Definition The PV01 of a bond is defined as the present value impact of a 1 basis point (0. 01%) increase (or “ bump”) in the yield curve. In the derivation below, we will refer to a generic “ discount curve”. As noted earlier, this discount curve, from which are derived the discount factors for the bond pricing calculation, can itself be considered as the sum of two curves: the “ underlying” yield curve (normally Libor), and a credit curve (reflecting the risk over and above the interbank risk ncorporated in the Libor curve). The PV01 calculates the impact on the price of bumping the underlying yield curve. Calculation For simplicity, consider the case of a zero coupon bond i. e. where there is only one cash flow, equal to the face value, and occurring at maturity in n years. Note, though, that the principles of the following analysis will equally apply to a coupon paying bond. We start by defining: P = price or present value today R(t) = discount rate, today, for maturity t FV = face value of the bond Then, from the above, we know:

P = FV/(1+r(t))^n Now consider the impact a 1bp bump to this curve. The discount rate becomes: R(t) = R(t) + 0. 0001 The new price of the bond, Pb(t), will be: Pb = FV/(1+[r(t)+. 0001])^n Therefore, the sensitivity of this bond to a 1bp increase to the discount curve will be: Pb – P = FV/(1+[r(t)+. 0001])^n - FV/(1+r(t))^n Eqn. 1 The first term is always smaller than the second term, therefore: * if we hold the bond (long posn), the PV01 is negative * if we have short sold the bond (short posn), the PV01 is positive We can also see that: the higher the yield (discount rate), the smaller the PV01. This is because a move in the discount rate from, for example, 8. 00% to 8. 01% represents a smaller relative change than from 3. 00% to 3. 01%. In other words, the higher the yield, the less sensitive is the bond price to an absolute change in the yield * the longer the maturity, the bigger the PV01. This is more obvious - the longer the maturity, the bigger the compounding factor that is applied to the changed discount rate, therefore the bigger the impact it will have.

To extend this method to a coupon paying bond, we simply note that any bond can be considered as a series of individual cash flows. The PV01 of each cash flow is calculated as above, by bumping the underlying yield curve at the corresponding maturity. In practice, where a portfolio contains many bonds, it would not be practical, nor provide useful information, to have a PV01 for every single cash flow. Therefore the cash flows across all the positions are bucketed into different maturities. The PV01 is calculated on a bucketed basis i. e. by calculating the impact of a 1bp bump to the yield curve on each bucket individually.

This is an approximation but enables the trader to manage his risk position by having a feel for his overall exposure at each of a series of maturities. Typical bucketing might be: o/n, 1wk, 1m, 2m, 3m, 6m, 9m, 1y, 2y, 3y, 5y, 10y, 15y, 20y, 30y. Worked example: Assume we hold $10m notional of a zero-coupon bond maturing in 7 years and the yield to maturity is 8%. Note that, for a zero coupon bond, the YTM is, by definition, the same as the discount rate to be applied to the (bullet) payment at maturity. We have: Price, P = $10m / (1. 08)^7 = $5. 834m

Bumping the curve by 1bp, the " bumped price" becomes: Pb = $10m / (1. 0801)^7 = $5. 831m Therefore, the PV01 is: Pb – P = $5. 831m - $5. 835m = -$0. 004m (or -$4k) Meaning In the example above, we have calculated the PV01 of the bond to be -$4k. This means that, if the underlying yield curve were to increase from its current level of 8% to 8. 01%, the position would reduce in value by $4k. If we assume the rate of change in value of the bond withrespectto the yield is constant, then we can calculate the impact of, for example, a 5bp bump to the yield curve to be 5 x -$4k = -$20k.

Note, this is only an approximation; if we were to graph the bond price against its yield, we wouldn't see a straight line but a curve. This non-linear effect is called convexity. In practice, while for small changes in the yield the approximation is valid, for bigger changes, convexity cannot be ignored. For example, if the yield were to increase to 9%, the impact on the price would be -$365k, not -(8%-9%)x$4k = -$400k. Use The concept of PV01 is of vital day to day importance to the trader. In practice, he manages his trading portfolio by monitoring the bucketed yield curve exposure as expressed by PV01.

Where he feels the PV01 is too large, he will perform a transaction designed to either flatten or reduce the risk. Similarly, when he has a view as to future yield curve movements, he will position his PV01 exposure to take advantage of them. In this case, he is taking a trading position. ii)CS01 The basis of the CS01 calculation is identical to that of the PV01, only this time we bump the credit spread rather than the underlying yield curve. The above example was based on a generic discount rate. In practice, for any bond other than a risk free one, this rate will be combination of the yield curve together with the credit curve.

At first glance therefore, we would expect that, whether we bump the yield curve or the credit spread by 1bp, the impact on the price should be similar, and described by Eqn. 1 above. What we can also say is that, bumping the yield curve, the overall discount rate will increase and therefore, as for PV01: * if we hold the bond (long posn), the CS01 is negative * if we have short sold the bond (short posn), the CS01 is positive From the same considerations as for PV01, we can see that: * the higher the credit spread, the smaller the CS01 * the longer the maturity, the bigger the CS01

In practice, when we look at multiple cash flows, the impact of a 1bp bump in the yield curve is not identical to a 1bp bump in the credit spread. This is because, inter alia: * the curves are not the same shape and therefore interpolations will differ * bumping the credit spread affects default probability assumptions that will, in turn, impact the bond price In general though, PV01 and CS01 for a fixed coupon bond will be similar. The exception is where the bond pays a floating rate coupon. In this case, the sensitivity to yield curve changes is close to zero so, although the PV01 will be very small, the CS01 will be “ normal”.

Worked example: A worked example would follow the same steps as for PV01 above, only this time we would bump the credit spread by 1bp rather than the underlying yield curve. Theta and Carry We now look at the two sensitivities arising from the passage of time (“ 1 day decay”, to use option pricing terminology). First, let’s calculate what the total impact on the value of a position would be if the only change were that one day had passed. In particular, we assume that the yield and credit curves are unchanged. Again, for simplicity, consider the case of a zero coupon bond i. . where there is only one cash flow, equal to the face value, and occurring at maturity in n years. Again, we note that the principles of the following analysis will equally apply to a coupon paying bond. Following the previous notation, the value (or price) today will be: P(today) = FV/(1+r(t))^n The value tomorrow will be: P(tomorrow) = FV/(1+r(t-1))^(n-1/365)Eqn. 2 There are two differences between the formula for the value today and that for tomorrow. Firstly, the discount rate has moved from r(t) to r(t-1). Here, r(t-1) is the discount rate for maturity (t-1) today.

We have assumed that the discount curve does not move day on day, therefore the rate at which the cash flow will be discounted tomorrow is the rate corresponding to a one day shorter maturity, today. Secondly, the period over which we discount the cash flows has reduced by one day, from n to n-1/365 (we divide by 365 because n is specified in years). Theta and Carry capture these two factors. P(tomorrow) – P(today) gives the full impact on the price due to the passing of one day. This impact can be approximated by breaking down the above formula into its two component parts i. e. he change in discount rate and the change in maturity, as explained below. iii)Theta As before, we define: P = price or present value today r(t) = discount rate, today, for maturity t FV = face value of the bond In addition, we define: r(t-1) = discount rate, today, for maturity t-1 (e. g. for a bond with 240 days to maturity, if the 240 day discount rate today is 8. 00% and the 239 day discount rate today is 7. 96% then: r(t) = 8. 00% and r(t-1) = 7. 96%) We now define Theta as: FV/(1+r(t-1))^n – FV/(1+r(t))^n We can see that, compared to the formula for the full price impact above (Eqn.), this sensitivity reflects the change in the discount rate but ignores the reduction by 1 day of the maturity. In other words, Theta represents the price impact due purely to the change in discount rate resulting from a 1 day shorter maturity but ignores the impact on the compounding factor of the discount rate resulting from the shorter maturity. Note that the sign of Theta, in contrast to PV01 and CS01, can be both positive and negative. This is because r(t-1) can be higher or lower than r(t), depending on the shape of the yield curve.

That said, in practice, given that yield curves are normally upward sloping, we would expect r(t) to be higher than r(t-1). Therefore Theta will normally be positive. In the same way, if the yield curve is flat, then Theta will be zero. iv)Carry Using the standard notation, we define Carry as: FV/(1+r(t))^(n-1) – FV/(1+r(t))^n Comparing to the formula for the full price impact above (Eqn. 2), we see that this sensitivity reflects the change in maturity on the compounding factor to be applied to the discount rate but ignores the impact on the discount rate itself of moving one day down the curve.

In other words, Carry represents the price impact due purely to the change in discount factor resulting from a 1 day shorter compounding period but ignores the impact on the discount rate resulting from the shorter maturity. Where discount rates are positive (r(t) > 0), Carry will always be positive since the first term will be larger than the second. Using the Taylor expansion, we can obtain a simplified approximate value for Carry. Remembering that: 1/(1+x)^n = 1 – n. x + (1/2). n. (n-1). x^2 - … we have: Carry = FV. 1-(n-1/365). r(t)) – FV. (1-n. r(t)) = FV. r(t). 1/365 Note that r(t). 1/365 would represent one day's " interest" calculated on an accruals basis since, in the case, the yield equals the coupon rate. (Note, where a position is accounted for on an accruals basis, and therefore valued at par, the yield will always equal the coupon.) In other words, this definition ties in to the intuitive idea of carry that we have from, say, a deposit where the carry would be equal to one day's interest, based on its coupon.

We can also see that Carry is directly proportional to the yield. We have now seen that, between them, Theta and Carry attempt to capture the two components affecting the price move arising from the passing of 1 day, all other factors being kept constant. There will be certain " cross" effects of the two that will not be captured when performing this decomposition. In other words, Theta + Carry will not exactly equal the full impact (as per Eqn. 2). The difference, however, will not normally be material.

In general, for a long bond position, both Theta and Carry will be positive as, with the passing of one day, not only will the annualised discount rate be less (reflecting the lower yield normally required for shorter dated instruments) but the compounding factor will be smaller (reflecting the shorter maturity). Worked example: Assume we hold $10m notional of a zero-coupon bond maturing in 240 days and the yield to maturity today is 8%. Also, the yield today for the 239 day maturity is 7. 96%. Theta = $10m/(1. 0796)^(240/365) - $10m/(1. 08)^(240/365) = $23, 159 Carry =

$10m/(1. 8)^(239/365) - $10m/(1. 08)^(240/365) $20, 047 Theta + Carry = $43, 205 To compare, the full price impact of a 1 day " decay" is: $10m/(1. 076)^(239/365) - $10m/(1. 08)^(240/365) = $43, 113 Summary We have now analysed the key sensitivities that explain the 1 day move in a bond's mark to market value. To summarise some of the main features; for a long bond position:

PV01 / CS01: negative for a fixed coupon or zero coupon bond, PV01 and CS01 will be similar the higher the yield/credit spread, the smaller the PV01/CS01

the longer the maturity, the bigger the PV01/CS01 for a floating rate coupon (with a Libor benchmark), PV01 will be very small but the CS01 will be " normal" Theta positive the flatter the curve, the smaller the Theta Carry * positive * proportional to the yield Extension to interest rate swaps In essence, all the above applies equally to interest rate swaps (IRSs) when calculating/explaining daily P&L. We start by noting that an IRS is simply the exchange of two cash flows, one fixed and one floating. Extending the analysis we made for bonds, we can say: a) The PV01 of the floating rate leg will be close to zero. This is as noted for a floating rate bond.

In both cases, as the yield curve changes so do the expected future cash flows but, at the same time, so will the discount rates at which they are PV'd. The two effects will broadly cancel out. (The PV01 will not be exactly zero because, once the Libor fixing occurs, the next cash flow becomes fixed and therefore effectively becomes a zero coupon bond, on which there will be PV01.) The fixed leg is similar to the fixed coupon stream on a bond and can be considered as a series of zero coupon bonds. Therefore the exact same analysis as applied to bonds above will apply to the fixed leg. An IRS that ays floating and receives fixed will have a PV01 sensitivity similar to that of a long bond position. c) IRSs are normally interbank trades where it is assumed that there is no credit risk over and above Libor. Therefore, the CS01 will be zero. Theta and Carry may be either positive or negative.

Appendix 1 : Date Conventions There are several methods for computing the interest payable in a period and the accrued interest for a period. A particular method applied to a transaction can affect the yield of that transaction and also the payment for a transaction. Counting the Number of Days

The conventions used to determine the interest payments depend on two factors: 1) The number of days in a period and 2) The number of days in a year. The conventions are: 0 Actual/360 1 Actual/365 : sometimes referred as Actual/365F (seldom used now) 2 Actual/Actual 3 30/360 European: sometimes referred to as ISMA method (30E/360) 4 30/360 US (30U/360) The first three methods (Actual/360, Actual/365 and Actual/Actual) calculate the number of days in a period by counting the actual number of days. For each method the number of days in a year is different. Actual/365 and Actual/Actual are similar except: 1.

Periods which include February 29th (leap year) count the number of days in a year as 365 under Act/365 and 366 under Act/Act; 2. Semi-annual periods are assumed to have 182. 5 days under Act/365 and however many actual days under Act/Act. Eurobond markets use the 30E/360 basis. This calculation assumes every month has 30 days. This means that the 31st of a month is always counted as if it were the 30th of the month. For 30E/360 basis, February is also assumed to have 30 days. If the beginning or end of a period falls on a weekend the coupon is not adjusted to a good business day.

This means that there are always exactly 360 days in a year for all coupons. For example a coupon from 08-November-1997 to 08-November-1998 of 5% is a coupon of 5%, even though 08-November-1998 is a Sunday. There is no adjustment to the actual coupon payment.

Appendix 2 : Calculating Accrued Interest Even though Eurobond coupons are not adjusted for weekends and holidays, the accrual of a coupon for any part of the year has to use the correct number of days. The difference between European and US 30/360 method is how the end of the month is treated. For US basis the 31st of a month is treated as the 1st of the next month, unless the period is from 30th or 31st of the previous month.

Euromoneymarkets: 0 Day count basis: actual/360 1 Settlement basis: spot (two day) standard 2 Fixing period for derivatives contracts: two day rate fixing convention Euro FX markets 3 Settlement timing: spot convention, with interest accrual beginning on the second day after the deal has been struck 4 Quotation: ‘ Certain for uncertain’ (ie 1 Euro = x foreign currency units) U. S.

In the case of Bunds, the day-count convention is the Act/Act convention. Appendix 1 contains more details of date conventions - it is recommended that you read this at the end of the module. The part of a year between the settlement date (27 July 2001) and the next coupon (4 February 2002) is: Day Count 192/365 (ie Actual days/Actual days) = 0. 5260 The price of the first coupon can therefore be calculated in the following way: PV of First

Coupon = 4. 8873% All of the other cash flow present values are calculated in the same manner. Adding them up gives us the price of the bond.

Accrued interest is calculated from 04 February 2001 to 27 July 2001 (173 days) : Accrued Interest Accrued = 5% x 0. 47397 = 2. 3699% There is more detail on Accrued interest in Appendix 2. It is recommended that you read it at the end of this module. Notice that the quoted price of the bond (the ' clean price') is 102. 2651% not 104. 6350% (which is the ' dirty price' or invoice price - ie the price actually paid for the bond). The dirty price is the sum of the present values of the cash flows in the bond. The price quoted in the market, the so-called " clean" price or market price, is in fact not the present value of anything.

It is only an accountants' convention. The market price, or clean price, is the present value less accrued interest according to the market convention. Practitioners find it easier to quote the clean price because it abstracts from the changing daily accrued interest (i. e. it avoids a " saw-toothed" price profile). This publication is for internal use only by Deutsche Bank Global Markets employees. The material (including formulae and spreadsheets) is provided foreducationpurposes only and should under no circumstances be used for client pricing.

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