Teaching system of linear equations

This essay is a critical analysis of Realistic Mathematics Education (RME) as a cognitive involvement of teaching and learning system of linear equations. The pedagogy will apply on year 7 students. My learning objectives for 12 years old students are to introduce a system of linear equations in an informal way. Since according to National Curriculum of Mathematics algebra starts in Key stage 3 at year 7 and system of equations is mostly taught in year 8 and year 9 but I would like to teach the students, to prepare them for the future so that it will be easy for them. To accomplish this objective I will also use the example from the Unit 'Comparing Quantities' of 'Mathematics in Context' (MiC) textbook. The main goal of this study is to transform the knowledge of equations in such a way that pupil " understand equations" and then applying it, rather than " doing equations" without understanding it. Freudenthal (1973) declared mathematics as a human activity, so by employing context from real world we will endeavor to achieve our learning aims.

Gender of the participant:

I have selected my 15 students in the class of mixed gender and majority of them are boys and there are only 5 girls in the class. As Forgasz (2006) argues that girls are better academically over boys in mathematics and Arnold (1997) states that " there may be differences in the development of the function of brain hemispheres, and that language development follows a biological maturation 'timetable', where girls have a faster rate of progress than boys." (Library, 2001, p. 1) The same is the case here in our mathematics class, in all formative and summative assessments the achieved results of girls are always better than boys. Since the number of

students is few in my class therefore I can easily focus on every student to achieve my targets.

Social background of the participant:

All students are belonging to entirely different cultural backgrounds, hailing from Somalia, India, Pakistan, China and Great Britain. So I can say that my class represents multicultural group. Two of the Somalian students have just migrated from their country so they have severe language problem and mostly rest of the students also face the same problem because English is not their home language. Most of my students belong to the working class families. Their social background is not strong. Vast opportunities of learning are not available to them. They can't access the facilities like tutoring.

Algebra and its background

- Mason (1996) defines algebra and states that

" The word algebra is derived from the problems of al-jabr (literally, adding or multiplying both sides of an equation by the same thing in order to eliminate negative/fractional terms), which were paralleled by problems of al-muqabala (subtracting the same thing from or dividing the same thing into both sides)." (Wessels, 2009, p. 16)

Traditional way of teaching and learning algebra has been a big factor in demise of its popularity. Lack of involvement of students during the class is a common picture. It has been observed that it has reached such heights that students dislike algebra. In worst scenario they even are scared of it. Bertrand Russell writes in his autobiography

" The beginning of Algebra I found far more difficult, perhaps as a result of bad teaching. I was made to learn by heart: The Square of the sum of two numbers is equal to the sum of the squares increased by twice their product. I had not the vaguest idea what this meant, and when I could not remember the words, my tutor threw the book at my head, which did not stimulate my intellect in any way". (Kooij, 2001, p. 1)

At present, the situation is not different, still the students are passing their GCSE without proper understanding of algebraic tools. In the result, they have a totally wrong perception of algebraic tools in the rest of their lives, and they are become the part of a formal traditional system. Algebra, which is a representative mathematical language, must be graspable for students as it is of vital importance. Being capable of solving algebra can lead the students to manage learning advanced. Thinking logically and analyzing accurately is a skill that a good mathematician can develop easily. Mathematics teaching should promote the pupils' ability to solve problems in many spheres . The algebra comprises of following main divisions consisting of variables, relations, function, equations and in equations (inequalities), and graphs.

The introduction of system of equations

" Equations are mathematical statements that indicate equality between two expressions." (Wessels, 2009, p. 18). System of linear equations means more than one linear equations having more than one variable. However, here our objective is not solving the formal system of equations; our objective is only to develop a system of equations through comparing quantities and relate them through real life context. So that students can feel comfort while

manipulating the system of equation. It has been observed by the researches that majority of the middle schools students face problems in understanding and solving the system o f linear equations (Eric J. Knuth & Ana C. Stephens's et al., 2006).

Teaching algebraic equations seems to have been a coaching problem since Ancient Times (Radford, 2000). Numerous researches is being conducted on algebraic equations but most of them are still in process to make it easy to understand for the pupils but still system of equations is considered a hard topic among the students .(Wijers, 2001)

Misconception of Equal sign:

Before introducing the simultaneous equations, there are some issues and problems, which need to be discussed. The first and very serious problem in year 7 students in misconception of equal sign '='. Since in National Curriculum of Mathematics algebra starts at year 7 and at this stage students have no proper concept of algebraic symbols and the common problem which is found in the investigation is the wrong perception of equal sign '=', As Stephens et al. (2006) also argues that " many of the difficulties that students have when working with symbolic expression and equations may be attributed to their misconceptions about the meaning of the equal sign." (p. 299). Mexicia (2008) also states that a common mistake among a student is misunderstanding the equal sign as a signal for doing something, instead of taking that sign as an operator. Generally, for students, the equal sign flashes a button in their minds to start working on something rather than its own entity as a " symbol of equivalence" or " quantity sameness".

Alibali et al. (1999) argue that " many elementary and middle school students demonstrate inadequate understanding of the meaning of the equal sign, frequently viewing the symbols as an announcement of the result of an arithmetic operation rather than as a symbol of mathematical equivalence." (Stephens et al., 2006, p. 298). He further explains that it was found by Falkner et al. (1999) that when they gave an " example from six grade students' i. e. 8 + 4 = â-¡ +5 then many students provided answers of 12, 17, or 12 and 17 - answers that are consistent with an understanding of equal sign as announcing a result, students added all the numbers in the equation or added all the number before the equal sign, again indicating an operational view of the equal sign" (Stephens et al., 2006, p. 298).

Utilization in daily life:

The real purpose of algebra teaching is to facilitate the students to use algebraic symbols or tools to help in solving the real life problems (Wessels, 2009). The use of system of equations in our daily life is very common. System of equations can be used to compare any two values. But before finding the values, it is necessary for the students to have pilot knowledge of all algebraic operations before manipulating the system of linear equations. Majority of the students at this level have problems to manipulate the system of linear equations. " Graeber & Tanenhaus (1993) says that numerous studies have concluded that that the students did not realize the situation is modeled in the problem, when students are working on making the algebraic equation, so they select the operation by guessing, by trying all operations and choosing one that gives what seems to them a reasonable answer (Schifter, 1999, p. 63) . This can be acceptable at junior level

students during solving system of equations but in senior classes it shows the lack of understanding in system of equations.

Realistic Mathematics Education:

The development of the RME (Realistic mathematics education) evolved after thirty years of developmental research in teaching and learning mathematics in the Netherlands and is primarily based on Freudenthal's interpretation of mathematics as a human activity (Freudenthal, 1973)

Past Review

Realistic Mathematics Education (RME) was the Dutch effort to change and reform the teaching of Mathematics all around the world in 1970s. It was developed by using extensive research and mainly applies the ideas of three mathematicians Freudenthal , Treffers and Gravemeijer (Autumn, 2003). The contemporary structure of RME is mostly determined by Freudenthals views on mathematics.

Two of its major points are

Mathematics must be connected to reality

Mathematics should be perceived as a human activity.(Zulkardi, 2002).

Freudenthal views on the method of mathematization were that pupils should be involved in guided reinvention of mathematics, in other words the pupils should develop their mathematical thinking and knowledge themselves same as the mathematicians did before them, starting with informal strategies and then gradually moving to more formal strategies(Carol Marshall, 2003), so RME context is used to help students to

understand Mathematics, which is in keeping with Freudenthals theory about mathematics that it must be close to the real life. Moreover, the purpose of making it valuable for children, mathematics should be related to the children and ultimately be reflecting the society (Carol Marshall, 2003). Later, Treffers (1978, 1987) gave the idea of two types of mathematization in an educational context and differentiated " horizontal" and " vertical" mathematization (Zulkardi, 2002) Freudenthal (1991) divides the horizontal and vertical mathematization and describes them as:

" Horizontal mathematization leads from the world of life to the world of symbols. In the world of life one lives, acts (and suffers); in other one symbol are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflecting: this is vertical mathematization. The world of life is what is experienced as reality (in the sense I used the word before), as is symbol world with regard to abstraction". (Fauzan, 2002, p. 39)

Characteristics or Tenets of RME

By summarizing three Van Hiele's levels and Treffer's modern mathematization and Freudenthal's didactical phenomenology, Zulkardi(2002) describes five fundamental features of Realistic Mathematics Education.

a) The use of contexts.

b)The use of models.

c) The use of students' own productions and constructions.

d) The interactive character of the teaching process

e) The intertwinement of various learning strands. (Zulkardi 2002)

RME'S KEY PRINCIPLES

According to Gravemeijer (1994) for instructional design there are three key principles of RME namely.

- guided reinvention through progressive mathematization,

- Didactical phenomenology

- Self developed models or emergent models.

Guided reinvention through progressive mathematization:

According to the first principle of RME , pupils should be given the chance to practice the same process by which mathematics was invented and a learning route has to be mapped out that help the pupils to find the intended mathematics by themselves and the reinvention of mathematical structure creates when the students start to use the daily routine language to make contextual problems into well organized and formal mathematics shape. (Armanto, 2002)

'Mathematizing' is a main movement in RME and this activity mainly involves generalizing and formalizing (Gravemeijer, 1994). Treffers(1987) defines the process of mathematization and states that formalizing includes modeling, symbolizing, schematizing and defining, and generalizing is to understand in a reflective sense and by solving the contextual problems in realistic approach students learn to mathematize contextual problems. (Fauzan, 2002)

De Lange (1996) described the process of conceptual mathematization in RME and states that the process of developing mathematical concepts and ideas starts from the real world, and at the end we need to reflect the solution back to the real world and in other words, in mathematics education things are taken from the real world , are methamitised and then brought back to the real world. (Fauzan, 2002)

Didactical phenomenology:

Freudenthal (1983) presents the thought of didactical phenomenology which means that learning mathematics should be starting from the real life contexts which are meaningful for the pupils, and it will motivate the students in learning processes. (Fauzan, 2002).

Self developed models:

The third key principle by Gravemeijer (1994) for instructional design in RME is self developed models. This principle plays an important role in making a connection between informal knowledge and formal knowledge. According to it, we have to give the opportunity to the students to use and develop their own models when they are solving the problems. To begin with the students will develop a model which is familiar to them. After the process of generalizing and formalizing, the model gradually becomes an entity on its own (Armanto, 2002).

Gravemeijer (1994) describes " this process a transition from model-of to model-for. After the transition, the model may be used as a model for mathematical reasoning" (Fauzan, 2002, p. 42)

Misconception of 'realistic'

There is a misconception about RME that its context is always from real life, it is not necessary always; it might be some imaginary stories and ideas to draw the concentration of the students towards the contextual problem. Hadi, (2002) defines that real world is the world which is outside the mathematics, it may be your school, your class or house or anything outside the mathematics. Gravemeijer (1999) explains the meanings of reality and states that.

'The use of the label 'realistic' refers to a foundation of mathematical knowledge in situations that are experientially real to the students. Context problems in RME do not necessarily have to deal with authentic every-day life situations. What is central, is that the context in which a problem is situated is experientially real to students in that they can immediately act intelligently within this context. Of course the goal is that eventually mathematics itself can constitute experientially real context for the students.'(Fauzan, 2002, p. 42)

Realistic mathematics education is not only associate mathematics with real life, it has proper rules and principles under which we have to teach students and develop their cognitive level . According to Feudenthal (1973) students should not be a passive receiver; they should be a part of learning through contextualizing process for their cognitive development. " In RME 'real world' is the starting point for the development of mathematics concept and ideas" (Hadi, 2002, p. 32). Learning mathematics means doing mathematics, of which solving everyday life problems (contextual problems) is an essential

part (Armanto, 2002). Freudenthal (1971) explains this activity, which is performed in RME is:

An activity of solving problems, of looking for problems, and also an activity of organizing a subject matter. This can be a matter from reality, which has to be organized according to mathematical patterns if they have to be solved. It can also be a mathematical matter, new or old results, of your own or others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach. (Fauzan, 2002, p. 34)

The concept of equal sign in RME:

As we have discussed that one of the issues related to an algebraic system of linear equations is the misunderstanding of equal sign. Nicole M. McNeil (2006) also argues that clear concept of equal sign is necessary for understanding the algebraic linear equation and it has been proven by the researches that if students have proper understanding of equal sign then they have a more possibility to solve the system of linear equations correctly. The misunderstanding of the students can be resolved by using RME, because as Zulkardi (2002) states that the first characteristic of RME is the use of context so it can be possible that students can clear the concept of equal sign through context from real life, Also Alibali et al.(2005) emphasis on the educator to give more " attention to the contexts and formats in which they are presenting problems because small differences in how problems are presented can influence what students come to understand about the associated concepts."(NICOLE M. MCNEIL et al. , 2006, p. 383). Furthermore, Nicole M. (2006) put more stress on the teachers' professional

development and changes in mathematical curricula and improving the teaching methods to convey the proper concept of equality sign in algebraic equations from contexts.

To overcome this problem some curriculums " try to offer students some help to develop the algebraic ideas and to acquire and make sense of signs. For example, in the new Ontario Curriculum of Mathematics (Ministry of Education and Training 1997), students are introduced to a kind of 'transitional' language prior to the standard alphanumeric- based algebraic language and are asked to find the value of ' * ' in equations like: * + * + 2 = 8 or the value of ' â-¡ ' in equations like 32 +â-¡ + â-¡ = 54. This 'transitional language' approach, as any pedagogical approach for teaching algebra, relies on specific conceptions about what signs represent and the way in which the meaning of signs is elaborated by the students".(Radford, 2000, p 239)

The concept of basic arithmetic operations in variables and constants in RME:

During solving the system of linear equations students have some misconception about the basic arithmetic operations in constant and variable . it can remove by suing RME technique because the basic thing in RME is the context through real life. Coral Marshall(2003) also states that with RME teaching the teacher or instructor can use the context and easily solve the students' misconception in algebraic operations.

Since in RME the first step is horizontal mathematization, and to achieve the first target the educator needs to make a realistic environment because

according to Freudenthal's didactical phenomenology realistic situation is very important to start the development of mathematical concepts. Realistic situation can be developed by drawing pictures on the white board or making some stories or physically selects some students to act. (Heeley, 2008) demonstrate the students by an excellent way to differentiate between variable and numbers and their addition. He puts some numbers and some bananas in a jar and invites a boy to find the variable and add them similarly the numbers, he can easily find out the bananas from the jar and separately add them, so this is the way to clear the concept of students in variable and numeric numbers' confusion. There are a lot of other methods to get the attention of students in the class and enhance the student's ability towards mathematics and according to Freudenthal (1973) students should be the active receivers not passives like other traditional classes . This is the best way to increase the cognitive level in the students learning. Kozulin (2003) is also in the favor of using symbolic tools in learning and states that " symbolic tools have a rich educational potential, but they remain ineffective if there is no human mediator to facilitate their appropriation by the learner" (p. 35) . So teacher's professional development is very necessary in teaching and learning through RME and it will be discuss later.

The role of RME for solving systems of equation:

As we have discussed above that real context enhance the student's ability and the learning aims of this essay is how to create a system of linear equation with informal and pre-formal methods using the scheme RME. Corel (2003) present some examples of exchanging goods to develop the concept

of equality and how to use the system of equations in students' mind. Corel explains her personal experience of using RME and finds a positive result in students' progress. She fulfills the first principle of RME and creates a context through real life by using an imaginary story in picture form and presents it in front of students. In the example, it is given that a farmer has two sheep and one goat. He wants to sell them and buy some bags of wheat but when he goes to the market he finds that he can exchange his one goat by six chickens and same for one sheep and one bag of salt is equal to two chickens. Similarly, three bags of salt can be exchanged by two bags of wheat.

According to RME rules after creating the realistic context, convert the problem in mathematical symbols which is called horizontal mathematization. At this step students should have some basic knowledge of the problem. For example here they should have some knowledge of equality sign and variables (as we have already discussed these issues). So first she introduce equal sign in algebra and named those things in algebraic notation say c for chicken , s for a bag of salt , h for sheep , g for goat and w for wheat. So according to the question

1g = 6c & 1h = 6c & 2c = 1s & 3s = 2 w

These are four equations which are found by given data. Till now, students able to understand how to make the simple equation. Now the next step solves the problem in mathematical symbols, which is called vertical mathematization. At these step students need some supervision of any instructor or teacher .

So altogether the farmer can exchange his goat and sheep by 18 chicken. For 18 chicken, we need to multiply both sides of third equation by 9 , because this is the rule of algebraic equation if you multiply any number on one side of equality you have to multiply the same number on the other side as well . So the third equation will become.

9 * 2c = 9 * 1s

18 c = 9 s

As he has now 18 chickens, so they are equal to 9 bags of salt . Now multiply fourth equation by 3 to get the answer i. e.

3s * 3 = 2w * 3

9 s = 6 w

It means the farmer can buy six bags of wheat when he exchanges his one goat and two sheep.

In this above example, the students can learn three things in a basic algebraic system of equation. The first thing is to understand the concept of equal sign and as second how to convert the real context in to an algebraic term (like variables) and third how to solve the system of equation to find the required result.

The realistic approach Vs the mechanistic approach

In the conventional (mechanistic) style of the learning process the teachers take control over each activity. In contrast, the RME approach suggests that the pupils is supposed to take responsibility for their own mathematical

learning, and they should be actively engaged in interactive discussion in the classroom. Guided by the teacher, the pupils reinvents informal and formal mathematics models in a process of mathematizing contextual problems (Armanto, 2002).

According to Heuvel- Panhuizen (1998) states that In the formal method of teaching mathematics, the learning subject matter is divided into insignificant small parts and the pupils are asked to solve these problems by fixed procedures and are trained by exercises, frequently to be done individually. This formal approach to mathematics education, which also rejects the mechanistic, is indeed a contrast to RME approach. RME has a more complex and meaningful conceptualization of learning. Active participation is expressed by students in a teaching-learning process, in which they established mathematical tools and understanding rather than being the receivers of ready-made mathematics.

In this view mathematics education would be highly interactive in which the teachers would have to build upon the ideas of the students. It means they have to react based on what the students bring to the fore (Kooj, 1999).

There is only one step more in RME from other formal methods of teaching mathematics and this is context from real life. It is often observed that in formal teaching methods the teacher straight away starts teaching mathematical problems without any introduction from context of a real world as freudenthal (1973) declares mathematics as human activity and should be connected to real life , so every problem in mathematics should be related to the real life and before starting the questions the educator should

be creating the environment from context of real life and then present it in front of students which in unfortunately not happen in our formal methods of teaching . " The jump to the formal level is made too quickly, and there is almost no time for students to develop their own schemes. The traditional algebra course is seen as sterile, disconnected from another mathematics and the real world" (Reeuwijk, 1995, p. 1)

Difference between word problem and RME:

When we are talking about RME, the first questions comes in our mind about RME , is this same like a word problem? Hadi (2002) describes that in solving a word problem it is necessary for the students to follow the precise rules and regulation. Moreover, students apply the symbols without reflecting the specific context.

The contextual problems in RME help in concept formation by providing the students an incentive and drive them with a natural approach towards understanding mathematics. These are also provide a fundamental principle and foundation for learning the mathematical procedures, symbols, and rules linked with the other models which act as important aid for thinking.

Solving system of equations through Comparing Quantities:

Since our objective is to familiarize system of equations to the year 7 students by using some informal ways. Now we are going to discuss those informal strategies for learning system of linear equation and to fulfill our aims we are using some schemes from the unit 'Comparing Quantities' of Mathematics in Context (MiC) textbook. These schemes were used by

ancient Greeks and Chinese's to solve a system of linear equations presented by Reeuijk (1995), Autumn (2003) and Kooij (2001).

These are some basic strategies to develop algebraic thinking . They make a bridge between student's previous arithmetic's knowledge and algebra. By applying these strategies students can easily understand the tools of making system of linear equations. Reeuijk (1995) called these strategies a tool which is used to make a link between the concrete and the abstract. All schemes are the basic informal strategies for manipulating the system of linear equation from the chapter 'Comparing Quantities' of Mathematics in a Context (MiC) textbook. An important aspect is that in unit Comparing quantities the ideas are usually presented as a story and thus the idea is to encourage the pupils to solve the problems in their own ways i. e. developing their own strategies. It has been noticed that by giving the problems in a natural context, students try to use the ideas that they may not have learned by their teachers at school. At this level more emphasis is on pre-formal methods and no formal algorithms are used because this unit is created for the 11-12 years old students who start algebra for the first time. The more formal shape of simultaneous equations is used in next chapters of the textbook for the next classes. The scheme (Reuwijeck , 1995) mentioned four main strategies __ guess and check , reasoning , combination charts and notebook. . Reeuwijk (1995) presents an example is his paper and in the textbook of MiC, in which a diagram of two T-shirts and two drinks is given and their price is 44 $. Similarly, in another diagram the price of 30$ is given for one T-shirt and three soda water. Students are asked to find the price of one soda bottle and one T-shirt. This question is looking like a typical word

problem, but as we discuss above that the difference between simple word problem and RME is the realistic context and here the context is in the form of pictures. The 'guess and improve' strategy is commonly used by students who have just basic know-how of the subject. Mostly low cognitive level students attempted this question by using this strategy.

The second strategy is reasoning through exchanging, this scheme was used by ancient Chinese . It is same like barter system, Students took a bit more interest in this strategy . The other two schemes are notebook notations and combination charts. These schemes are mostly used by the students having a high cognitive level. All above schemes are horizontal mathematization. According to Gravemeijer (1994) realistic environment is very important in the progression of mathematical concept. First, there are many problems in the world that are required to be solved , later pupils apply their mathematical tools to resolve them realistically(Reeuwijk, 1995)FIGURE_5

Reeuwijk(1995) states that once student endeavor to solve the problems by their own informal ways, it was summarized together (see fig. 1) and students realize that all schemes are interrelated . Now the next step in vertical mathematization. In which it is taught by the teacher to manipulate the system of equations and solve it in formal way.

Fig1 progressive formalization

(Kooij, 2001, p. 143)

It is necessary to change the traditional way of teaching system of equations and use context from real life to make the proper concept of the topic in

students' mind. Our dilemma is that algebra starts at KS3 in year 7, so students have no base of algebra before year 7 but at this stage if we provide the proper understanding to the students than they will never consider algebraic equations as a hard topic in their future. Reeuwijk (1995) presents the above methods to create the sense of simultaneous equations in the early stage of about 11 years old. He uses these strategies and got a very positive result.

According to Reeuwijk (1995) if we leave the students to allow them making their own ways to solve the problem with their previous knowledge of mathematics than they will learn more because students will use their mind more and becomes the active participants of the class and can easily understand algebraic term. Moreover " the non-statutory guidance to the National Curriculum states that pupils at every stage should be encouraged to develop their own methods and pupils need to have opportunities to develop informal, personal methods of recording and an approach in which pupils are required to use and apply their developing knowledge and skills leads to more effective learning"(Antumn, 2003, weblink) as Freudenthal (1973) declares that the students should be an active receiver in the class , so in these way students will become an active participant in the solving any problem . Moreover, this thing will be helpful in their real life problem solving. " By discussing and reflecting on the students' use of strategies, they start realizing that formal strate