The derivatives are an imperative class finance essay

Finance

CHAPTER 1

Derivatives are an imperative class of financial instruments nowadays. The use and variety of derivatives have grown tremendously in the last few decades. They are said to be the main catalyst to the financial and trade markets nowadays as mentioned in (Mai, 2008), since the numerous types of risk protection are offered in derivatives, where they also allow some kind of innovative investment strategies. For instance, the development and growth of derivatives market in India, is relatively a recent spectacle. The market has exhibited exponential growth both in terms of volume and number of traded contracts since its initiation in June 2000. The market turn-over has grown from Rs. 2365 crore in 2000-2001 to Rs. 11010482. 20 crore in 2008-2009 (Vashishtha and Kumar, 2010). Derivatives are stated as financial instruments which derive their value from the value of an underlying security, group of securities or an index such as options and futures contracts. In general, it is defined as an agreement or a contract that has a value determined by the price of something else. According to Mai (2008) again, derivatives has experienced the most innovations comparing to other class of financial instruments, where its market has grown impressively around 24 percent per year in the last decade into a sizeable and truly global market with about €457 trillion of notional amount outstanding compared to a small and domestic derivatives market about 25 years ago. The growth is often due to an increase in price volatility in different markets. This was true in the oil market in 1973-1974 when Organization of the Petroleum Exporting Countries (OPEC) reduced the supply, and also in 2006 – 2007. Thus, it followed by high and variable oil prices. On the other hand, the U. S interest rates became more volatile following inflation and recession in the 1970’s. Deregulation of electricity and natural gas prices also led to price volatility and to the greater use of derivatives in these markets. There is a natural connection between the price variability and the development of derivatives markets, where it is unnecessary to manage risk when there is no risk (McDonald, 2006). Some may argue that derivatives have engaged as a significant role in the near collapses or bankruptcies of Barings Bank in 1995, Long-term Capital Management in 1998, Enron in 2001, Lehman Brothers and American International Group (AIG) in 2008. They are even regarded as time bombs for the economic system and financial weapons of mass destruction (Chui, n. d). However, with appropriate and accurate management of derivatives, it could lead to substantial benefits towards the economy as a whole. These financial instruments unsurprisingly, could help those economic users to improve their market and credit risk management. Moreover, with derivatives as the financial instrument, they will lead to a financial innovation and market development as mentioned before. The tasks of ensuring that these instruments transactions being properly traded and prudently supervised would be the main challenges for those who involved in the derivatives market. Derivatives can be looked at from various perspectives such as the end-users (the people or institution that enter into derivatives contracts for any of the reasons that will be mentioned later), the market-makers, traders or brokers (the people who buy and sell derivatives to the end-users where usually they aim to make profit for their services), and the economic observers (people who analyze the activities of the end-users and market-makers from a financial and mathematical point of view).

The main objective of derivatives is to minimize risk as in eliminating the uncertainty for one party while offering the potential for a high returns which at increased risk to another. This is called as a risk management or in other word, a concept of hedging. As mentioned in Millman (2008), the diverse range of potential underlying assets and payoff alternatives result in a large scale of derivatives contracts available to be traded in the market. Another function of derivatives is that it can be used as speculative purposes that can be constructed to make a bet on almost anything. Furthermore, without actually buying and selling stocks and bonds, sometimes derivatives can be used to obtain the same financial result that we would get from trading these instruments (Cherry and Gorvett, 2010). This can lower transaction costs such as broker's fees and bid-ask spread. Moreover, derivatives can be used to get cash for stock that we own without selling it immediately, allowing we to defer taxes on any gain and to avoid the risk of price changes. All these benefits have led to the extensive use of derivatives where approximately 92 percent of the world's 500 largest companies manage their price risks using derivatives (Mai, 2008). Options, alike forward and futures contracts are among the financial instruments under the derivatives market that have been used and traded so widely during the past decades. They are among the most important inventions of modern-day finance. Options give the holder or the owner the right, but not the obligation to buy or sell a particular asset or underlying stock at a fixed price for a specified period of time, whereas a futures contract commits one party to deliver, and another to pay for, a particular good at a particular future date (Weishaus, 2011). The fact that these instruments protect against loss in value but do not necessitate the hedgers to sacrifice potential gains is making them very attractive contracts to hedgers. Most exchanges that trade futures also trade options on futures although; there are other types of options in the market as well. For instance, in 1973 the Chicago Board of Trade established the Chicago Board Options Exchange to trade options on stocks, while the Philadelphia Stock Exchange has a flourishing business in currency options. These two and many other successful option markets arose from the revolution and innovation of the option pricing model introduced and developed by Fischer Black, Myron Scholes and Robert Merton in 1973 in their famous journal of political economy; " The Pricing of Options and Corporate Liabilities". Since their innovation on pricing method, many other subsequent studies and researches started to form Black-Scholes-Merton model as their basis and benchmark for pricing the option on their models. For example, Cox, Ross and Rubinstein (1979) introduced Binomial Trees Pricing Model, while Stutzer, Jackweth and Rubinstein (1999) introduced Jump-Diffusion and Bollerslev (1987) introduced Generalized Autoregressive Conditional Heteroskedasticity (GARCH). All these pricing models basically were using the Black-Scholes-Merton model as their start-up base and guide to develop and improve the original model to become more simplify and suitable with respect to their current issues, demands and economy. As a result, Fischer Black, Myron Scholes and Robert Merton won the 1997’s " Noble Prize" in economy for their discovery on new methods in determining the value of options and their contribution as path-breaking studies that have formed the basis of option pricing for many subsequent academic studies. For the past 40 or 50 years, many researchers have developed the option pricing model with respect to the risk-neutral probabilities as in risk-neutral world and real probabilities as in real world. Black and Scholes (1973) for example, assumed investors are risk-neutral and inferred the risk-neutral probabilities in their model. However, a few years later, they used the " instantaneous Capital Asset Pricing Model (CAPM)" derivation that allows for changing risk-adjusted discount rates. Unfortunately, the real world probability measure parameters felt out, leading to their Black-Scholes model being difficult to be explained with respect to the real world economy. There were few other models that have been using risk-neutral and real probabilities such as the Binomial Tree Option Pricing Model done by Cox, Ross and Rubinstein (1979) as mentioned before. They use the risk-free rate that yields their model, where it only employs the probability measure for the risk-neutral economy. On the other hand, Stutzer (1996) inferred real world probabilities densities from options data but only for the purpose of risk-neutral pricing not for real-world pricing. To conclude, we can see a lot of option pricing models infer either risk-neutral probabilities or real probabilities towards the risk-neutral world or real world pricing economy but each of the model has their own conceptual and practical problem that will be mentioned later on this paper.

Problem Statement

There have been some key issues in option pricing mainly on the problematic issues with option pricing models. Based on the analysis done by Hewett and Igolnikov (2000), where they examined the three traditional options pricing models as in Black Scholes method (BSM), Merton paradigm method and Binomial Tress of Cox, Ross and Rubenstein (CRR), they discovered that the main issue of pricing model was a misrepresenting the market microstructure. The underlying stock process as in their evolution and distribution said to play as an important role towards this issue. Notionally, only some of the stock follows Geometric Brownian Motion (GBM) since the rest of them might be in an inconsistent state towards the constant drift and constant volatility. Moreover, only a few assets return time series can be said as normally distributed, where others are not necessarily to be said so. Thus, not all prices are log-normally distributed. Gisiger (2010) on the other hand, stated that the concept of risk-neutral probability as assumed in Black-Scholes method is only valuable for a complete market and arbitrage-free pricing. A complete market is a market that does not have any transaction cost involve, that has no dividend payment involve in underlying security, and relates to a possibility of borrowing and lending cash at a known constant risk-free interest rate. This is an obvious counter-factual since in a real or actual economy world, the market usually is said to be an incomplete market where there must be some kind of transaction cost involve such as bid-ask spread. Plus, the principle of arbitrage is always practiced by investors where there is an opportunity to make a riskless profit and the market always relates to the involvement of dividend payment in underlying security, unlike the complete market. As stated by Murthy (2011), not all investors are risk-neutral as assumed in the traditional pricing models that have been mentioned before. Some of them are typically risk-averse and they desire to be compensated for bearing the risk as in risk premia unlike in Black Scholes and a few traditional methods that inferred the event probabilities from an economy that does not compensate risk bearing, although we are pricing assets from a real-world economy that does compensate risk bearing. Thus, the risk-neutral probabilities are design almost surely bound to be wrong as measures of real-world odds. This is also supported by " Credit Risk Puzzle" done by Hull, Pedrescu and White (2005), where they proved on their analysis that the fact of risk-averse investors requiring risk premia, is consistent with their result. There were some option pricing models that recognized these issues. For instance, Black and Scholes (1973) used the " instantaneous Capital Asset Pricing Model (CAPM)" derivation that allows for changing risk-adjusted discount rates, but as been stated earlier, the real world probability measure parameters felt out, making the Black-Scholes model hard to interpret with respect to the real world economy. Another example is The Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model by Bollerslev in 1986 that replaces the constant volatility with stochastic volatility. However, it still cannot incorporate the interpretation of the other probability measure parameters with respect to the actual world. Furthermore, Stutzer (1996), Jackweth and Rubinstein (1996), and Ait-Sahalia and Lo (1998) have been using different methods with different probabilities either in risk-neutral or real probabilities. They all have different goals where some infer risk-neutral probabilities towards risk-neutral pricing goal or real-world pricing, where others may even infer real probabilities towards the risk-neutral pricing goal. However, none of them have been inferring the real-world probabilities with respect to the real-world pricing goal by using the security’s actual discount rate that consider the incomplete market as in risk-averse function and real option analysis.

Objectives of Study

The objectives of this study are: To obtain real-world probability distributions from risk-neutral probability distribution with respect to the observed option prices, by using risk transformation methods. To compare the accuracy of real-world, risk-neutral and historical probability distributions by using the log-likelihood function for observed index levels. To compare the option prices (put and call options) in Binomial Tree Model and Black Scholes model with the observed option market prices by using real-world and risk neutral probability in order to determine which model gives the most accurate value when compare to the market price.

Scope and Limitation of Study

The scopes of the study are to obtain the real-world distributions from risk-neutral densities with respect to the observed option prices by using the risk transformation methods, measuring the accuracy of risk-neutral, real-world and historical densities by using the log-likelihood function and to compare the option price on both traditional methods as in Black-Scholes and Binomial Tree with respect to risk-neutral and real world distributions. The data consists of a historical time series of futures and options prices that based on the FTSE-100 index (Financial Times and Stock Exchange) that is traded in LIFFE (The London International Financial Futures and Options Exchange). Daily settlement values for futures prices and option implied volatilities were obtained from LIFFE, for the 126 consecutive expiry months from July 1993 to December 2003 inclusive. Option prices are available on average for 37 exercise prices. These strikes are nearly always separated by 50 index points. The dividend data and the risk-free interest rates, defined by the London Eurocurrency rate, were also obtained. The probability distributions that will be obtained in objective 1 can be used in put and call option to determine the price and for the purpose of this study, we only used the put and call option instead of any other exotic options in the process of comparison the option pricing model and the data for that process are based on the closing prices of Standard and Poor 500 Index. The data used consist of daily close prices from January 3, 2012 to June 22, 2012. The annual volatility is assumed to be 174 days instead of 365 days. The selection of this period of observation is based on a term close to the period considered by most investors since choosing an appropriate period of observation (n) is quite difficult due to changes of volatility and inaccuracy of data. All the data were collected from Bloomberg, Datastream and Yahoo Finance.

1. 5Significance of Study

The significances of this study are mainly toward the academic and market purposes. The study of comparison between option pricing models with respect to risk-neutral and real probabilities towards the risk-neutral and the real world pricing is very essential for the academic purpose. The main reason is because we are facing the incomplete market in the actual economy world where we must consider the real-world assumptions as in real probabilities, and not just accepting and assuming that the actual economy world is totally based on the risk-neutral assumptions. Thus, it will provide better understanding for economic observers, academicians, or even investors to know the differences between the real option with respect to real-world distributions that based on the actual economy and the risk-neutral option with respect to risk-neutral distributions that totally based on the risk-neutral economy. While for the market purpose, the risk-averse investors have a more opportunity to make a wise decision to invest in real option instead of risk-neutral option, since the real option infers real-economy that does compensate risk bearing as in risk premia and provide the probability of success of a real-option project. This is good for market since we know there are many types of investors exist in the market, where it is basically not only based on risk-neutral investors. Furthermore, investors will be able to make a better judgment since this paper will measure the accuracy of each probability, density and distribution towards the historical data and the mean absolute error (MAE) method is used for each of the pricing models that consist risk-neutral and real probabilities to determine which model gives the most accurate or the closest value to the market price.

CHAPTER 2

LITERATURE REVIEW

2. 1Introduction

First and foremost, the literature review discusses on the overview of option and option pricing; what the option is all about, what the benchmark for option pricing is, and what the common problems related to the option pricing models. Next, is the discussion of risk neutral probabilities (RNP) and real probabilities (RP), which it covers the characteristic for each of the probability, the differences between them, the advantages and problems, the previous studies that have been using these probabilities and the issues arise from all these previous studies. Afterward, further discussion is on the methodology part, where we are focusing on the methods that will be used to obtain real-world distributions from risk-neutral densities and how to measure the accuracy of them. Also we will be covering the traditional pricing models as in Black-Scholes and Binomal Tree Model where we will discuss on the reasons of choosing these models as our benchmark towards our objectives. Under this sub-topic also, it covers the framework of these and other pricing models that have been used in previous studies, and the issues of risk neutral and real probabilities that arise from all these models. Last but not least, the literature discusses the issues on Mean Absolute Error (MAE) and Root Mean Square Error (RMSE), since we want to find available statistical measures of the average inaccuracy for each of the pricing model so that we will determine which model is the most accurate when compared to market price.

2. 2Option and Option Pricing

An option can be defined in many words from a long interpretation to the simplest one but with the same basic understanding. Black and Scholes (1973) stated in the simplest form, that option is a security that gives the right to buy or sell an underlying asset, based on specified conditions and within a specified period of time. While Cox, Ross and Rubinstein (1979) stated that an option is a security which gives its owner the right to trade in a fixed number of shares of a specified common stock at a fixed price at any time on or before a given date. Whereas Hemmings (1993) mentioned that an option is a contract between two parties, say party A and B, where party A grants party B the right to buy a given asset from or sell to A, at B’s decision at a fixed price until a fixed date after which any right or obligations expire. Main advantage of options is that they enable the holder or owner to benefit from upside potential while limiting downside risk (Obiyathulla, 2012). There were some path-breaking articles that have formed the basis of option pricing for many subsequent academic studies. It started with a revolutionary change in 1973, where Black and Scholes model was presented as the first completely satisfactory equilibrium option pricing model. Merton (1973) extended their model in various important ways during that same year. In 1997, Fischer Black, Myron Scholes and Robert Merton were presented as the " nobel prize" winners in economics for their new methods in determining the value of options. As being shown by these studies, the concept of option pricing is applicable to almost every area of finance, where the theory applies to a very general class of economic problems. Hewett and Igolnikov (2000) analysed the key issues in option pricing mainly on the problematic issues with option pricing models. They examined on three traditional option pricing methods as in Black Scholes method (BSM), Merton paradigm method and the binomial trees of Cox, Ross and Rubenstein (CRR). They found out a misrepresenting the market microstructure was a main issue of the pricing model. The evolution and distribution of the underlying stock process played as an important role towards this matter. According to their analysis, theoretically not all underlying stock follows Geometric Brownian Motion (GBM), where some of them are intermittent with the constant drift and constant volatility as assumed in the GBM. A " pull to par" of a bond and electricity prices were among the examples mentioned that were inconsistent with the GBM assumptions. Furthermore, most of the assets return time series are not normally distributed and as a result, most prices are not necessarily log-normally distributed. They examined the valuation of an interest rate swaption where it is considered as a good example of a mixture of models that has some kind of complex inconsistency. Based on their method, they found out that the difference in the valuation of log-normal and normal model was quite high, as stated around US$183, 000. 00. Another issue that has been mentioned was the actual implementation of " market accepted" methodology. Binomial trees are one of the examples. Plenty different ways of implement the binomial trees are exist in the markets that are used to value a variety of financial mechanism. The variation of the final result depends on the number of times used to create the tree. In order to converge to a stable result, different number of time steps is needed for every implementation, where each of the implementation contains a function of many parameters. They showed a comparison of valuation results of an interest rate swaption, produced by two different software programs, utilizing the same (" normal") methodology. The notional amount of the transaction was US$100, 000, 000. 00. They concluded that the proliferation of implementation schemes and custom-modified valuation models has made standardization impossible.

2. 3Risk Neutral Probabilities (RNP) and Real Probabilities (RP)

Risk-neutral probabilities are the probabilities of future outcomes adjusted for risk, which are then used to compute the expected asset values. The benefit of this risk-neutral pricing approach is that the once the risk-neutral probabilities are calculated, they can be used to price every asset based on its expected payoff. These theoretical risk-neutral probabilities differ from actual real-world probabilities; if the latter were used, expected values of each security would need to be adjusted for its individual risk profile. According to Gisiger (2010), there has been some confusion regarding the understanding of the risk-neutral probability in the two angles, said to be Financial Economics' perspective and Mathematical Finances' perspective. The prices of the primary securities themselves are assumed to be given by the markets and are not explained accurately by mathematical finance. Therefore the concept from the financial economics' perspective must be explored and understood deeply. Based on his analysis, he strongly stated that the concept of risk-neutral probability is only valuable for a complete market and arbitrage-free pricing, where there is an existent of unique system state price that leads to a unique risk-neutral probability measure. The market is said to be complete and states prices are unique, if and only if the number of states is not more than the number of securities with linearly independent payoffs. These statements are also supported by Constantinides, Jackwerth and Perrakis (2008). According to Gisiger (2010) again, on his assessment towards the risk-neutral probabilities by using arrow securities system illustrations that consist of a linear combination of arrow securities, stochastic payoff and redundant security, he found out that a lot of frequent statements about risk neutral probabilities were identified as misconceptions. As a result, he specified that a risk-neutral is neither the real probability of an event happening nor an independent of the real event probability. However, the measures of both risk-neutral probability and real probability are said to be parallel given that both of them agree on the possible and the impossible outcomes. The Girsanov Theorem and Radon-Nikodym derivative have been used as the measures for obtaining risk-neutral probability that was based on a continuous-time model, which follow continuous stochastic processes. He concluded that if the market participant is ready to engage in arbitrage activities, a unique arbitrage-free price only serves as a trading criterion. Otherwise, it is still impractical since the market participant however, might have different views regarding the risk premia, where a unique arbitrage-free is dependent of risk premia. He also stated that an arbitrage-free price is not necessarily a fair price, or in other word the correct price, but it is just only a market consistent price. Constantinides, Jackwerth and Perrakis (2008) added that there have been some kinds of restrictions on the arbitrage-free principle when the market is imperfect or incomplete. One of the restrictions is the prices of options that are too weak to be useful, either for confronting the data with a testable hypothesis or for option pricing. Thus, they presented an integrated approach of the option pricing that allows for imperfect and incomplete markets. They introduced the economic restriction that at least one risk-averse trader is a marginal investor in the options and the underlying asset. The implied restrictions have been tested by simply solving a linear program. In addition, the testable restriction on option prices that include the Black Scholes Model as a special case was provided. The S&P 500 index options traded on the Chicago Board Options Exchange have been tested. As a result, they found out that the economic restrictions were violated regularly. From that result, they concluded that, the Black Scholes Model does not incorporated with the market incompleteness and realistic transaction costs and cannot be completely ascribed to the mispricing of these options (S&P 500 index options). Similar study on the risk-neutral probabilities and real probabilities has been done by Murthy (2011), where he focused on the valuation of credit risk and the role of default probabilities. According to him, by exploiting just one assumption, the real-neutral probabilities can be obtained clearly. The assumption is that investors are considered to be risk-neutral and desire no risk premia. Based on his study, he stated that since some investors are typically risk-averse and their desire compensation for bearing the risk, the assumption of risk-neutral is obviously counter-factual. He added that the risk-neutral probabilities as measures of the real-world odds are design almost certainly bound to be wrong. Supported by " Credit Spread Puzzle" (see Table 1 below) done by Hull, Pedrescu and White (2005), in their sample, risk-neutral probabilities of default average rating classes were shown as 5. 7% per year and exceeded the average annual actual probabilities of default of 3. 7% per year. Their data consists of yield spreads on 7-year corporate debt from 1996 to 2004 and historical default frequencies from Moody’s over 1970 to 2003. With this result, they argued that the fact of risk-averse investors requiring risk premia, is consistent with their result.

Table 1: Real-world and risk-neutral default intensities (1 basis point is 0. 01%)

RatingReal-world default Intensity per year (bp)Risk-neutral default Intensity (bp)RatioDifferenceAaa46716. 863Aa67813. 072A131289. 8115Baa472385. 1191Ba2405072. 1267B7499021. 2153Caa and lower169021301. 3440Table 1 shows that the ratio of the risk-neutral to real-world default intensity decreases as the credit quality declines. However, the difference between the default intensities increases as credit quality declines. The size of the difference between the two default intensity estimates is sometimes referred to as the credit spread puzzle. Risk-neutral pricing may cause a few problems, practically and conceptually, as suggested by Arnold and Crack (2003). They said that practical issues with risk-neutral pricing would happen when inferred option pricing parameters are not relevant to the real world. The probabilities in the real and risk-neutral worlds are different, such as the probability of default on a corporate bond, the probability that an American-style option will finish in-the-money, at-the-money or out-of-the money, the probability of success of a real-option project and the likelihood of a jump in a jump process are each different in the real and risk-neutral worlds. As indicated by them, practical problems would also arise if higher moments such as skewness and kurtosis contributed in the asset pricing model, since the higher moments and variance can vary between the real and risk-neutral worlds. Thus, it will lead to a difficulty of pricing. As they believe, conceptual problems on the other hand arise, because it is difficult to explain the need of the event probabilities from an economy that does not compensate risk bearing, although we are pricing assets from a real-world economy that does compensate risk bearing. Cox et al. (1985) also supported these problems and explained that these issues could be avoided by performing the real-option analysis using the probability distributions of the real-world economy or in other word, using the real probabilities instead of risk-neutral probabilities. According to Arnold and Crack (2003) again, there are three advantages of modelling the option pricing by using the real-world probabilities instead of the risk-neutral world probabilities. The first one is that the related actual-world probabilities are implemented directly through the model, for instance, of success in a real-option project, of default on corporate bond or of an American-style option finishing in the money. Secondly, the managerial anxiety of practitioners that is generated when they use the risk-free discount rates for risky cash flows while they are competing risk-neutral models would be avoided. Thirdly, when higher moments such as skewness and kurtosis appear in asset pricing models, the actual-world probabilities model, unlike the risk-neutral probabilities model, would simplify the pricing of an option.

2. 4Risk-Transformation Methods and Density Estimation Method

In recent years, attention has shifted to the relationship between risk-neutral and real-world densities. Options prices and risk assumptions are said to have a big influence towards the derivation of risk-neutral and real-world densities. As proven by Breeden and Litzenberger (1978), a unique risk-neutral density for the possible values of a subsequent asset price can be inferred from European put or call prices when contracts are priced for all strikes and when there are no arbitrage opportunities. Transformations from a risk-neutral density to a real-world density can be derived by making assumptions about risk preferences. An ironic source of information about the future distribution of the underlying asset price, is said can be provided by option prices, particularly with respect to future volatility. By assuming either a parametric family, or a flexible shape defined by spline functions, initially the information can be translated into risk-neutral densities. A risk-transformation is needed, since we want to move from an artificial risk-neutral world to the real world that compensates risk bearing as in risk premia (Arnold and Crack, 2003). Therefore, a parametric utility function is chosen and the relative risk aversion will be estimated which is assumed to be constant. Risk-neutral densities also can be calibrated statistically, to obtain a second set of real-world densities from option prices. As supported and suggested by Ait-Sahalia, Yacine and Lo (2000), by transforming the parametric, all the real-world densities could be derived, thus the risk-neutral densities will have closed forms and can be calculated swiftly.

2. 5Traditional Option Pricing Models: Black-Scholes and Binomial Tree

A stochastic risk-adjusted discount rate is required by the continuous-time option pricing model under the real-world probability measure, where no single risk-adjusted discount rate will do the job. The binomial tree option pricing model done by Cox, Ross and Rubinstein (1979), does not incorporated with the real-world economy, since they use the risk-free rate that yields their model, where it only employs the probability measure for the risk-neutral economy not the actual or real-world economy. They do not provide enough information needed to price options in the real world. However, information on how to deduce real-world option pricing was adequately provided, where they stated that the underlying security’s actual discount rate employs the probability measure for the actual economy or the real-world economy. Black and Scholes (1973) on the other hand, also recognized this issue. Thus, they used the " instantaneous Capital Asset Pricing Model (CAPM)" derivation that allows for changing risk-adjusted discount rates. Unfortunately, the parameters of the real world probability measure fell out and as a result, their Black-Scholes model was difficult to interpret with respect to the real world economy. Later, the Black-Scholes model was improved to pact with some limitations of the real world. The Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model for instance, replaces the constant volatility with stochastic volatility. However, with this improvisation, they still cannot incorporate the interpretation of the other probability measure parameters with respect to the actual world. Stutzer (1996) suggested that when other things being equal, as the stock price increase (decreases), the degree of leverage implicit within a call option on that stock decreases (increases), and thus the risk-adjusted discount rate for the call option also decreases (increases). It follows that the risk-adjusted discount rate for the option that applies over the entire life of the option is a path-dependent random variable driven by the randomness of the stock price. Nevertheless, he used diffusions instead of binomial trees, historical data, and the subjective density to estimate the risk-neutral density for risk-neutral pricing, whereas we wanted to price in the real-world economy. According to Jackweth and Rubinstein (1996), both real and risk-neutral probabilities can be inferred from the option prices by using the binomial trees although they use risk-neutral probabilities in their model. They also used nonparametric techniques that require large data sets. As a result, it was still didn’t incorporate with the real world, where the absence of trading costs was very crucial. Ait-Sahalia, Yacine and Lo (1998) also infer probabilities densities from option prices by using the risk-neutral densities, not the real-world ones. Moreover, with the use of the same nonparametric techniques and jump-diffusions as mentioned before, it is still somehow violated the actual world, since they don’t consider the implication for preference and risk aversion function. All these statements bring to robust conclusions. First, binomial model is an approximation to an assumed real-world diffusion or pure jump process where binomial model framework allows to use real-world moments and to avoid the use of utility functions. Furthermore, by using the security's actual discount rate, we can employ probability measure for the actual or real-world economy since we know a continuous-time option pricing model under the real-world probability measure requires a stochastic risk-adjusted discount rate. In order to do that, we can apply a risk-transformation method to obtain real-world distribution from the risk-neutral densities and estimate the parameters for the purpose of measuring the accuracy, as mentioned before. Even though binomial model is the closest approximation to an assumed real-world economy, it still does not provide the real-world probability that the product will be profitable However, the probabilistic information about the actual economy could be gained, by setting the model firmly in the real world as in making the real-option analysis more lucid to skeptical managers. Experienced management may have a good estimate of the actual probability of success of a project being valued as a real-option.

2. 6Mean Absolute Error (MAE) and Root Mean Square Error (RMSE)

There are a few methods to find available statistical measures of the average inaccuracy and reexamine the relative abilities of two, dimensioned measures of average model performance error such as Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). Willmott and Matsuura (2005) examined both of these methods with a set of model-produced estimates, to explore and interpret available statistical measures of the average inaccuracy. With five cases of hypothetical testing, and a scatterplot of 10 pairs of MAE and RMSE, they found out that the interpretation of RMSE is confounded because there is no consistent functional relationship between RMSE and average error (see Table 2 below). Plus, the distribution of error magnitude increasingly larger than MAE with n^1/2 as it becomes more variable, since its lower limit is fixed at MAE and its upper limit (n^1/2*MAE) increases with n^1/2. In other word, RMSE can be said as one of special interest because it is widely reported in the climatic and environmental literature. Nevertheless, it is an appropriate and misinterpreted measure of average error. Measures of average error such as RMSE (that are based on sum of squared errors), are functions of the average error (as in MAE). Consequently, these measures do not interpret the average error alone. Furthermore, since there is no clear interpretation of RMSE, they recommended that such measures no longer be reported in the literature, and all previous model-performance assessments and inter-comparison should be reassessed and debatable, if those were based largely on RMSE or related measures. They concluded that MAE is more natural and appropriate measure of average error, where it is also a more conspicuous measure of average error magnitude compared to RMSE. Supported by Nau (2005), the MAE is also measured in the same units as the original data, and is usually similar in magnitude to, but slightly smaller than, the RMSE. He concluded that with their simplest form of calculation, the mathematically challenged find MAE is an easier statistic to understand than the RMSE or any other error method since it can effortlessly be interpreted to non-academics. Thus in conclusion, we have decided that MAE is the most suitable error method to be used in this paper.

Table 2: Five hypothetical sets (cases) of 4 errors, and their corresponding totals, MAEs and RMSEs. Each ( ) is a hypothetical error value.

VariableCase 1Case 2Case 3Case 4Case

52110021100231102357888888MAE222221620285064RMSE2. 02. 22. 63.

54. 0Table 2 shows the error-magnitude variance increase steadily from Case 1 (where it is zero) through Case 5 (where it is at its maximum). RMSE also increases steadily. It is apparent that the lower limit of RMSE is MAE, which occurs only when . It is easily shown that the upper limit of RMSE is . It is clear then that RMSE varies with the variability of the error magnitudes (or squared errors), as well as with the total-error or average-error magnitude (MAE) and.

CHAPTER 3

METHODOLOGY

3. 1Introduction

This chapter will emphasize the description sample, data collection, and the methods that will be used to reach our objectives as mentioned before. Furthermore, the flow of methodology will be provided to show and summarize every single step of the methodology process. First, by using Risk-Transformation Methods, the real-world distribution will be obtained from risk-neutral densities with respect to the observed option prices. Then, the transformation parameters will be estimated by using the Log-Likelihood

Functions and the accuracy of risk-neutral, real world and historical densities with respect to those parameters will be compared. Next is the estimation of the volatility by using the Simple Moving Average (SMA). Later would be the valuation of option prices (call and put options) by using the Black-Scholes and Binomial Tree model with respect to these real-world and risk-neutral distributions. Last but not least, would be the comparison of these pricing models with the observed market price, by using the Mean Absolute Error (MAE).

3. 2Sample of Data

The data consists of a historical time series of futures and options prices that based on the FTSE-100 index (Financial Times and Stock Exchange) that is traded in LIFFE (The London International Financial Futures and Options Exchange). Daily settlement values for futures prices and option implied volatilities were obtained from LIFFE, for the 126 consecutive expiry months from July 1993 to December 2003 inclusive. Option prices are available on average for 37 exercise prices. These strikes are nearly always separated by 50 index points. The dividend data and the risk-free interest rates, defined by the London Eurocurrency rate, were also obtained. For the process of comparison the option pricing model, the data will be based on the closing price of Standard and Poor 500 Index. The data used consist of daily close prices from January 3, 2012 to June 22, 2012. The annual volatility is assumed to be 174 days instead of 365 days, which will be used in Simple Moving Average calculation. The selection of this period of observation is based on a term close to the period considered by most investors since choosing an appropriate period of observation (n) is quite difficult due to changes of volatility and inaccuracy of data. All the data were collected from Bloomberg, Datastream and Yahoo Finance.

3. 3Flows of Methodology

Figure 1: Flows of Methodology

3. 3. 1Risk-Transformation Methods

A unique risk-neutral density for the possible values of a subsequent asset price can be inferred from European call prices or put prices when contracts are priced for all strike prices and there are no arbitrage opportunities. The risk-neutral density (RND) is then:(3. 3. 1. 1)(3. 3. 1. 2)With the continuous risk-free rate and the time remaining until all option expire. With representing the risk-neutral measure:(3. 3. 1. 3)Furthermore, the forward price for time is the risk-neutral expectation of(3. 3. 1. 4)The forward price is also a future price when simple and well-known assumptions are made about interest rates and dividend payments. Transformations from a risk-neutral density go to a real-world density can be derived by making assumptions about risk preferences. The first transformation considered involves a representative of utility function. With sufficient assumptions, the risk-neutral density, the real-world density and the utility function are linked by: (3. 3. 1. 5)The second transformation makes use of the calibration function of a miss-specified density. Let denotes the risk-neutral, cumulative distribution function (c. d. f) of, and then define . The calibration function is the real-world c. d. f of the random variable For real-world probabilities, the c. d. f. of is:(3. 3. 1. 6)Consequently, the real-world density is given by:(3. 3. 1. 7)The two transformations are equivalent whenever defines a marginal utility function.

3. 3. 2Density Estimation Method: The Log-Likelihood Function

To estimate a real-world density from option prices, we require an estimate of the risk-neutral density and an estimate of either a marginal utility function or a calibration function. We consider two parametric specifications of and parametric functions for and which are all chosen to enable us to derive closed-form, real world densities . It is the stage that contains the estimation of the risk aversion parameter y and the calibration parameters j and k. These estimates do not vary across time, unlike the estimated RND (risk-neutral density) vectors which vary across a set of months

i = 1, 2, 3, …. n

Let be the set of n times when the chosen option contracts expire and let be the time when RNDs (risk-neutral densities) are formed for options that expire at time . We ensure that densities do not overlap, that is:(3. 3. 2. 1)with . With denoting the observed price of the underlying asset at time , the log-likelihood function then equals:(3. 3. 2. 2)Here denotes the transformation parameter(s), either or (j, k). This function is maximized to provide the two-step maximum likelihood estimate of. The log-likelihood of a set of RNDs is obtained when there is no transformation, so either Likewise, the log-likelihood of non-overlapping real-world densities obtained from time series of asset returns is also given by summing the logarithms of density values.

3. 3. 3Simple Moving Average (SMA)

SMA is the unweighted mean of the previous daily return (n) data points. We define as a continuously compounded return during t (between at the end of day t-1 and at the end of day t by taking the logarithm of current asset value, divided by the value of the day before, . Day to Day Price Changes in a Market:(3. 3. 3. 1)Where:= value of the market variable at the end of day t= value of the market variable at the end of day t – 1Average Day-to-Day Changes over a Certain Period:(3. 3. 3. 2)The Usual Estimation of the Standard Deviation,, of the (assuming n = 174 trading days per year, as mentioned in our scope and limitation before):(3. 3. 3. 3)(3. 3. 3. 4)(3. 3. 3. 5)

3. 3. 4Black-Scholes Model

Since we want to compare the Black-Scholes model and Binomial Tree model with respect to risk-neutral and real probabilities, we will show how to price the option on these models using our collected data. First is the traditional Black-Scholes model that inferred the risk-neutral probabilities. The Black-Scholes model can be used to value European call or put options either without dividend payments or with dividend payments. As mentioned before, the traditional Black-Scholes model assumes no dividend payment, constant volatility and follows the principle of no arbitrage. In this study, we will consider both European call and put option. Price of call option:(3. 3. 4. 1)Price of put option:(3. 3. 4. 2)Where:(3. 3. 4. 3)(3. 3. 4. 4)Where:= The price of the call or put option= The current price of the underlying stock= The underlying strike price (exercise price)= The continuous compounded risk-free interest rate= The dividend yield on the stock (assumed as in this model)= The volatility of the stock (assumed as constant in this model)= The time to maturity of the option= The standard cumulative normal distribution function

3. 3. 5Binomial Tree Option Pricing Model

When comes to the traditional Binomial Tree model, we are incorporating the restriction to only two possible prices that consistent with the name of " binomial" that follows a binomial distribution. Under this model it is assumed that at any point in time, the stock price can change to either an up value or a down value. In-between, greater or lesser values are not permitted. Again, we are only considering European call and put options for the purpose of this study. The modeling of the stock price evolution is as follow:(3. 3. 5. 1) (3. 3. 5. 2)(3. 3. 5. 3)(3. 3. 5. 4)Where:= One plus the rate of capital gain on the stock if it goes up (up value)= One plus the rate of capital loss on the stock if it goes down (downvalue)= The value of the call or put option when the stock goes up (the payoff)= The value of the call or put option when the stock goes down (the payoff)= The number of shares that replicates the option payoff= The amount of borrowing/lending that replicates the option payoff= The continuously compounded risk-free interest rate= The dividend yield on the stock (assumed as in this model)= The volatility of the stock= The length of the period; where is the time to maturity of the option and refers to the number of binomial periodsThe cost of creating the option is the net cash required to buy the shares and bonds. Under the binomial tree model, by mimicking the payoff to a call or put by buying shares and borrowing some value as stated as synthetic call or put, we will able to determine the call or put price. This is done by using the law of one price where the idea that says the positions that have the same payoff should have the same cost. Thus, the cost of the option is: The put option:(3. 3. 5. 5)The call option:(3. 3. 5. 6)Alternatively we can obtain the put option price by using the risk-neutral and real-world pricing concept as follows: Price of put and call option by using risk-neutral pricing concept:(3. 3. 5. 7)(3. 3. 5. 8)Price of put and call option by using real-world pricing concept:(3. 3. 5. 9)(3. 3. 5. 10)Where:(3. 3. 5. 11)is the risk-neutral probability of an increase in the stock price while is the real-world probability based on the real-world distribution obtained from the objective 1. In order to not violate the law of one price, there must be no arbitrage opportunity exists. Thus, the assumed stock price movements as in and , should not give rise to arbitrage opportunities. In particular, we require that:(3. 3. 5. 12)

3. 3. 6 Mean Absolute Error (MAE)

As mentioned in the literature review before, we have decided that Mean Absolute Error (MAE) is the most suitable error method to be used in this paper instead of Root Mean Square Error (RMSE) to measures the average inaccuracy, as we want to see which model is more accurate to the market option price. MAE is a quantity used to measure how close forecasts or predictions are to the eventual outcomes. In general, it is an average of the absolute errors; where is the actual or true value and is the prediction or estimation value that will lead to: As for our estimation, the formula is as follow:(3. 3. 6. 1)Where:= Pricing error for an option at time t= Market option price at time t= Estimation option price based on Black-Scholes model or Binomial Tree model with respect to risk-neutral and real probabilities distributions(3. 3. 6. 2)Where: n = The number of the lifetime of the volatility (total days used in this study as mentioned in our scope and limitation) = 174

EXPECTED RESULT

We expect both utility-transformed densities and calibrated densities from the risk-transformation methods will have less skewness and kurtosis than risk-neutral densities. However, both set of parametrics as in risk-transformation methods together with the risk risk-neutral densities may have higher likelihoods than observed historical densities that will be estimated in the density estimation method in the log-likelihood function. As a result, the parametric real-world densities that will be obtained from risk-neutral densities with respect to the observed option prices can be said are more informative that these obtained historical time series of the index. Thus, these distributions can be stated as strong predictive tools toward our third objective. We will note that flexible spline densities are less informative than the parametric densities for our data. Encompassing densities, defined by a combination of parametric, real-world densities and historical densities, will provide the most satisfactory predictive for the data investigated. For the process of valuation the volatility and the option prices, we expect that the option prices that are less volatile would be more appropriate models to be used where they have better accuracy when compare to other models and to the market prices. Based on our expectation, the value of option prices for all pricing models as in Black-Scholes and Binomial Tree with respect to risk-neutral and real distribution will not vary too much, where the difference could just be in very small percentage since risk-neutral and real probability doesn’t diverge too much. To conclude, we expect the Binomial Tree model with respect to real-world distribution will have slightly advantage in term of accuracy towards the option market prices, when compare to other pricing models, since the model is based on real-world distribution or in other word in actual economy, and the model itself is the closest approximation to an assumed real-world economy as mentioned and proven by many previous studies.