

Error correction model

[Science](#), [Statistics](#)



Introduction Exchange rates play a vital role in a country's level of trade, which is critical to every free market economies in the world. Besides, exchange rates are source of profit in forex market. For this reasons they are among the most watched, analyzed and governmentally manipulated economic measures. Therefore, it would be interesting to explore the factors of exchange rate volatility. This paper examines possible relationship between EUR/AMD and GBP/AMD exchange rates. For analyzing relationship between these two currencies we apply to co-integration and error correction model.

The first part of this paper consists of literature review of the main concepts. Here we discussed autoregressive time series, covariance stationary series, mean reversion, random walks, Dickey-Fuller statistic for a unit root test. * The second part of the project contains analysis and interpretation of co-integration and error correction model between EUR/AMD and GBP/AMD exchange rates. Considering the fact, that behavior of these two currencies has been changed during the crisis, we separately discuss three time series periods: * 1999 2013 * 1999 to 2008 * 2008 to 2013. -----

Autoregressive time series A key feature of the log-linear model's depiction of time series and a key feature of the time series in general is that current-period values are related to previous period values. For example current exchange rate of USD/EUR is related to its exchange rate in the previous period. An autoregressive model (AR) is a time series regressed on its own past values, which represents this relationship effectively. When we use this model, we can drop the normal notation of Y as the dependent variable and

X as the independent variable, because we no longer have that distinction to make.

Here we simply use X_t . For instance, below we use a first order autoregression for the variable X_t . $X_t = b_0 + b_1 X_{t-1} + \epsilon_t$

Covariance stationary series To conduct valid statistical inference we must make a key assumption in time series analysis: We must assume that the time series we are modeling is Covariance Stationary. The basic idea is that a time series is covariance stationary, if its mean and variance do not change over time. A covariance stationary series must satisfy three principal requirements.

- * Expected value of the time series must be constant and finite in all periods.
- * Variance should be constant and finite.
- * The covariance of the time series with itself for a fixed number of periods in the past or future must be constant and finite.

So, we can summarize if the plot shows the same mean and variance through time without any significant seasonality, then the time series is covariance stationary. What happens if a time series is not covariance stationary but we use auto regression model? The estimation results will have no economic meaning.

For a non-covariance- stationary time series, estimating the regression with the help of AR model will yield spurious results.

Mean Reversion We say that time series shows mean reversion if it tends to fall when its level is above its mean and rise when its level is below its mean. If a time series are currently at its mean reverting level, then the model predicts, that the value of the time series will be the same in the next period $X_{t+1} = X_t$. For an autoregressive model, the equality $X_{t+1} = X_t$ implies the level $X_t = b_0 + b_1 X_t$ or $X_t = b_0 / (1 - b_1)$

So the auto regression model predicts that time series will stay the same if its current value is $b_0/(1 - b_1)$, increase if its current value is below $b_0 / (1 - b_1)$, and decrease if its current value is above $b_0 / (1 - b_1)$. Random Walks A random walk is a time series in which the value of the series in one period is the value of the series in the previous period plus an unpredictable error. $X_t = X_{t-1} + \epsilon_t$, $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma^2$, $E(\epsilon_t, \epsilon_s) = 0$ if $t \neq s$. This equation means that the time series X_t is in every period equal to its value in the previous period plus an error term, ϵ_t , that has constant variance and is uncorrelated with the error term in previous periods. Note, that this equation is a special case of auto correlation model with $b_0 = 0$ and $b_1 = 1$. The expected value of ϵ_t is zero. Unfortunately, we cannot use the regression methods on a time series that is random walk. To see why, recall that if X_t is at its mean reverting level, then $X_t = b_0 / (1 - b_1)$. As, in a random walk $b_0 = 0$ and $b_1 = 1$, so $b_0 / (1 - b_1) = 0/0$. So, a random walk has an undefined mean reverting level. However, we can attempt to convert the data to a covariance stationary time series.

We create a new time series, Y_t , where each period is equal to the difference between X_t and X_{t-1} . This transformation is called first-differencing. $Y_t = X_t - X_{t-1} = \epsilon_t$, $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma^2$, $E(\epsilon_t, \epsilon_s) = 0$ for $t \neq s$. The first-differenced variable, Y_t , is a covariance stationary. First note, that $Y_t = \epsilon_t$ model is an auto regressive model with $b_0 = 0$ and $b_1 = 0$. Mean-reverting level for first differenced model is $b_0 / (1 - b_1) = 0/1 = 0$. Therefore, a first differenced random walk has a mean reverting level of 0. Note also the variance of Y_t in each period is $\text{Var}(\epsilon_t) = \sigma^2$. Because the variance and the mean of Y_t are constant and finite in each period, Y_t is a covariance

stationary time series and we can model it using linear regression. Dickey-Fuller Test for a Unit Root If the lag coefficient in AR model is equal to 1, the time series has a unit root: It is a random walk and is not covariance stationary. By definition all random walks, with or without drift term have unit roots. If we believed that a time series X_t was a random walk with drift, it would be tempting to estimate the parameters of the AR model $X_t = b_0 + b_1 * X_{t-1} + \epsilon_t$ using linear regression and conduct a t-test of the hypothesis that $b_1 = 1$. Unfortunately, if $b_1 = 1$, then x_t is not covariance stationary and the t-value of the estimated coefficient b_1 does not actually follow the t distribution, consequently t-test would be invalid. Dickey and Fuller developed a regression based unit root test based on a transformed version of the AR model $X_t = b_0 + b_1 * X_{t-1} + \epsilon_t$. Subtracting x_{t-1} from both sides of the AR model produces $x_t - x_{t-1} = b_0 + (b_1 - 1)x_{t-1} + \epsilon_t$ or $x_t - x_{t-1} = b_0 + g_1 x_{t-1} + \epsilon_t$, $E(\epsilon_t) = 0$ where $g_1 = (b_1 - 1)$. If $b_1 = 1$, then $g_1 = 0$ and thus a test of $g_1 = 0$ is a test of $b_1 = 1$. If there is a unit root in the AR model, then g_1 will be 0 in a regression where the dependent variable is the first difference of the time series and the independent variable is the first lag of the time series. The null hypothesis of the Dickey-Fuller test is $H_0: g_1 = 0$ that is, that the time series has a unit root and is non stationary and the alternative hypothesis is $H_a: g_1 \neq 0$, that the time series does not have a unit root and is stationary.

To conduct the test, one calculates a t- statistic in the conventional manner for \hat{g}_1 but instead of using conventional critical values for a t- test, one uses a revised set of values computed by Dickey and Fuller; the revised set of critical values are larger in absolute value than the conventional critical

values. A number of software packages incorporate Dickey- Fuller tests. REGRESSIONS WITH MORE THAN ONE TIME SERIES Up to now, we have discussed time-series models only for one time series. In practice regression analysis with more than one time-series is more common.

If any time series in a linear regression contains a unit root, ordinary least square estimates of regression test statistics may be invalid. To determine whether we can use linear regression to model more than one time series, let us start with a single independent variable; that is, there are two time series, one corresponding to the dependent variable and one corresponding to the independent variable. We will then extend our discussion to multiple independent variables. We first use a unit root test, such as the Dickey-Fuller test, for each of the two time series to determine whether either of them has a unit root.

There are several possible scenarios related to the outcome of these test. One possible scenario is that we find neither of time series has a unit root. Then we can safely use linear regression to test the relations between the two time series. A second possible scenario is that we reject the hypothesis of a unit root for the independent variable but fail to reject the hypothesis of a root unit for the independent variable. In this case, the error term in the regression would not be covariance stationary.

Therefore, one or more of the following linear regression assumptions would be violated; 1) that the expected value of the error term is 0. 2 that the variance of the error term is constant for all observations and 3) that the error term is uncorrected across observations. Consequently, the estimated regressions coefficients and standard errors would be inconsistent. The <https://assignbuster.com/error-correction-model/>

regression coefficient might appear significant, but those results would be spurious. Thus we should not use linear regression to analyze the relation between the two time series in this scenario.

A third possible scenario is the reverse of the second scenario: We reject the hypothesis of a unit root for the dependent variable but fail to reject the hypothesis of a unit root for the independent variable. In the case also, like the second scenario, the error term in the regression would not be covariance stationary, and we cannot use linear regression to analyze the relation between the two time series. The next possibility is that both time series have a unit root. In this case, we need to establish where the two time series are co-integrated before we can rely on regression analysis.

Two time series are co-integrated if a long time financial or economic relationship exists between them such that they don't diverge from each other without bound in the long run. For example, two time series are co-integrated if they share a common trend. In the fourth scenario, both time series have a unit root but are not co-integrated. In this scenario, as in the second and third scenario above, the error term in the linear regression will not be covariance stationary, some regressions assumptions will be violated, the regression coefficients and standard errors will not be consistent, and we cannot use them for the hypothesis tests.

Consequently, linear regression of one variable on the other would be meaningless. Finally, the fifth possible scenario is that both time series have unit root, but they are co-integrated in this case, the error term in the linear regression of one term series on the other will be covariance stationary. Accordingly, the regression coefficients and standard errors will be

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consistent, and we can use them for the hypothesis test. However we should be very cautious in interpreting the results of regression with co-integrated variables.

The co-integrated regression estimates long term relation between the two series but may not be the best model of the short term relation between the two series. Now let us look at how we can test for co-integration between two time series that each have a unit root as in the last two scenarios above. Engle and Granger suggest this test: if y_t and x_t are both time series with a unit root, we should do the following: 1) Estimate the regression $y_t = b_0 + b_1x_t + \epsilon_t$ 2) Test whether the error term from the regression in Step 1 has a unit root coefficients of the regression, we can't use standard critical values for the Dickey - Fuller test.

Because the residuals are based on the estimated coefficients of the regression, we cannot use the standard critical values for the Dickey- Fuller test. Instead, we must use the critical values computed by Engle and Granger, which take into account the effect of the uncertainty about the regression parameters on the distribution of the Dickey- Fuller test. 3) If the (Engle - Granger) Dickey- Fuller test fails to reject the null hypothesis that the error term has a unit root, then we conclude that the error term in the regression is not covariance stationary.

Therefore, the two time series are not co-integrated. In this case any regression relation between the two series is spurious. 4) If the (Engle-Granger) Dickey- Fuller test rejects the null hypothesis that the error term has a unit root, then we conclude that the error term in the regression is covariance stationary. Therefore, the two time series are co-integrated. The

parameters and standard errors from linear regression will be consistent and will let us test hypotheses about the long - term relation between the two series. .

If we cannot reject the null hypothesis of a unit root in the error term of the regression, we cannot reject the null hypothesis of no co-integration. In this scenario, the error term in the multiple regressions will not be covariance stationary, so we cannot use multiple regression to analyze the relationship among the time series. Long-run Relationship For our analysis we use EUR/AMD and GBP/AMD exchange rates with respect to AMD from 1999 to 2013 with monthly bases. After estimating the normality of these time series we found out that the normality has rejected.

We got right skewness result and to correct them we used log values of exchange rates. Studying the trade between Armenia and Europe or Great Britain we found out that there is almost no trade relationship between them. Besides we assume, that Armenian Central Bank keeps floating rate of AMD. Taking into consideration these two factors the impact of AMD is negligible to have an essential influence on EUR/GBP rate. That is why we assume that the next models we will build show the relation between EUR and GBP. Graph 1 represents movement of EUR/AMD ; GBP/AMD since 1999 to 2013.

From it we can assume that these two currencies have strong long run relationship until Global Financial Crisis. As a result of shock in 2008 the previous relationship has been changed. However, it seems to be long term co-movement between the currencies. To accept or reject our conclusions we examine exchange rates until now including Global Financial Crisis, <https://assignbuster.com/error-correction-model/>

without crisis and after crisis. Co-integration of period from 1999 to 2013 To be considered as co-integrated the two variables should be non-stationary. So the first step in our model is to check the stationarity of variables by using Augmented Dickey-Fuller Unit Root Test.

EViews has three options to test unit-root: * Intercept only * Trend and Intercept * None From the first graph it is visible, that the sample average of EUR/AMD time series is greater than 0, which means that we have an intercept and it should be included in unit-root test. Although, series goes up and down, data is not evolving around the trend, we do not have increasing or decreasing pattern. Besides, we can separately try each of the components and include trend and intercept, if they are significant. In the case of EUR/AMD the appropriate decision is only intercept. Table 1. 1Table 1. We see it from the Table 1. 1, where Augmented Dickey-Fuller test shows p-value of 0. 1809 and as we have decided to use 5% significance level, Null Hypothesis cannot be rejected, which means there is a unit root. So, EUR/AMD exchange rate time-series is non-stationary. The same step should be applied with GBP/AMD exchange rates. We have estimated it and found out, that Augmented Dickey-Fuller test p-value is 0. 3724, which gives us the same results, as in the previous one: the variable has unit root. Since, the two variables are non-stationary, we can build the regression model $y_t = b_0 + 1x_t + ? t$ (Model 1. 1) and use e_t residuals from this model. So, the second step is to check stationarity for these residuals. Here we should use Eagle Granger 5% critical value instead of Augmented Dickey Fuller one, which is equal to -3. 34. Comparing this with Augmented Dickey-Fuller t-Statistic -1. 8273. Here minus signs should be ignored. So, comparing two values, we

cannot reject Null Hypothesis, which means residuals have unit-root, they are non-stationary. This outcome is not desirable, which means the two variables are not co-integrated.

Co-integration till crisis period (1999-2008) Referring back to graph 1, we assume that in 1999-2013 time series two variables are not co-integrated because of shock related to financial crisis. That is why it will be rational first to exclude data from 2008 to 2013 and then again check co-integration between two variables. Here the same steps should be applied as in checking co-integration for time series from 1999 to 2013. For time series from 1999 to 2008, for EUR/AMD exchange rate, Augmented Dickey-Fuller test p-value is 0.068. From the p-value it is clear that we cannot reject Null Hypothesis, which means it has a unit root. Having unit root means EUR/AMD exchange rate time-series is non-stationary. Now we should test stationarity of GBP/AMD exchange rates. The Augmented Dickey-Fuller test p-value is 0.2556, which means the variable is non-stationary. Since, the two variables are non-stationary, we should build the regression model and using residuals check stationarity. Table 2.1 In the table above Augmented Dickey Fuller t-test is 3.57 and so greater than Eagle-Granger 5% significance level critical value 3.34. That is why we can reject Null Hypothesis and accept Alternative Hypothesis, which means that residuals in regression model has no unit root. Consequently, they are stationary and we can conclude, that EUR/AMD and GBP/AMD time series are co-integrated: have long run relationship. As the variables such as EUR/AMD and GBP/AMD are co-integrated, we can run the error correction model (ECM) as below $D(y_t) = b_2 + b_3 D(x_t) + b_4 U_{t-1} + V$ (Model 1.2) * $D(y_t)$ and $D(x_t)$ are first differenced variables b_2 is the

intercept * b_3 is the short run coefficient * V white noise error term * U_{t-1} is the one period lag residual of \hat{y}_t . U_{t-1} is also known as equilibrium error term of one period lag. This U_{t-1} is an error correction term that guides the variables of the system to restore back to equilibrium. In other words, it corrects this equilibrium. The sign before b_4 or the sign of error correction term should be negative after estimation. The coefficient b_4 tells as at what rate it corrects the previous period disequilibrium of the system.

When b_4 is significant and contains negative sign, it validates that there exists a long run equilibrium relationship among variables. After estimating Model 1. 2, short run coefficient value b_3 has been 1. 03 and was found significant. And b_4 , the coefficient of error term has been 5. 06 percent meaning that system corrects its previous dis-equilibrium at a speed of 5. 06% monthly. Moreover, the sign of b_4 is negative and significant indicating that validity of long run equilibrium relationship between EUR and GBP.

Co-integration during crises period (2008-2013) Now is the time to check stationarity of variables in the period after crisis by the same way as we did above. From the ADF test it is clear that the two variables are non-stationary, after which we can construct ADF ; Eagle Granger test for residuals. However, because of ADF t-statistic is smaller, than Eagle Granger critical value, we could not reject that the residuals have unit-root. So, they are non-stationary and co-integration does not exist between the two currencies.