

# The concept of probability in mathematics



Probability is a way of expressing knowledge or belief that an event will occur or has occurred. The concept has been given an exact mathematical meaning in probability theory, which is used extensively in such areas of study as mathematics, statistics, finance, gambling, science, and philosophy to draw conclusions about the likelihood of potential events and the underlying mechanics of complex systems.

The word probability does not have a consistent direct definition. In fact, there are two broad categories of probability interpretations, whose adherents possess different views about the fundamental nature of probability.

The word Probability derives from Latin word *probabilitas* that can also mean *probity*, a measure of the authority of a witness in a legal case in Europe, and often correlated with the witness's nobility. In a sense, this differs much from the modern meaning of probability, which, in contrast, is used as a measure of the weight of empirical evidence, and is arrived at from inductive reasoning and statistical inference.

History:

The scientific study of probability is a modern development. Gambling shows that there has been an interest in quantifying the ideas of probability for millennia, but exact mathematical descriptions of use in those problems only arose much later.

According to Richard Jeffrey, “ Before the middle of the seventeenth century, the term ‘ probable’ meant approvable, and was applied in that sense,

univocally, to opinion and to action. A probable action or opinion was one such as sensible people would undertake or hold, in the circumstances.”[4] However, in legal contexts especially, ‘probable’ could also apply to propositions for which there was good evidence.

Aside from some elementary considerations made by Girolamo Cardano in the 16th century, the doctrine of probabilities dates to the correspondence of Pierre de Fermat and Blaise Pascal (1654). Christiaan Huygens (1657) gave the earliest known scientific treatment of the subject. Jakob Bernoulli’s and Abraham de Moivre’s *Doctrine of Chances* (1718) treated the subject as a branch of mathematics. See Ian Hacking’s *The Emergence of Probability* and James Franklin’s *The Science of Conjecture* for histories of the early development of the very concept of mathematical probability.

The theory of errors may be traced back to Roger Cotes but a memoir prepared by Thomas Simpson in 1755 (printed 1756) first applied the theory to the discussion of errors of observation. The reprint (1757) of this memoir lays down the axioms that positive and negative errors are equally probable, and that there are certain assignable limits within which all errors may be supposed to fall; continuous errors are discussed and a probability curve is given.

Pierre-Simon Laplace (1774) made the first attempt to deduce a rule for the combination of observations from the principles of the theory of probabilities. He represented the law of probability of errors by a curve  $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$ ,  $x$  being any error and  $y$  its probability. He also gave (1781) a formula for the law of facility of error (a term due to Lagrange, 1774), but one which led to

unmanageable equations. Daniel Bernoulli (1778) introduced the principle of the maximum product of the probabilities of a system of concurrent errors.

The method of least squares is due to Adrien-Marie Legendre (1805), who introduced it in his *New Methods for Determining the Orbits of Comets*. In ignorance of Legendre's contribution, an Irish-American writer, Robert Adrain, editor of "The Analyst" (1808), first deduced the law of facility of error,

$h$  being a constant depending on precision of observation, and  $c$  a scale factor ensuring that the area under the curve equals 1. He gave two proofs, the second being essentially the same as John Herschel's (1850). Gauss gave the first proof which seems to have been known in Europe (the third after Adrain's) in 1809. Further proofs were given by Laplace (1810, 1812), Gauss (1823), James Ivory (1825, 1826), Hagen (1837), Friedrich Bessel (1838), W. F. Donkin (1844, 1856), and Morgan Crofton (1870). Other contributors were Ellis (1844), De Morgan (1864), Glaisher (1872), and Giovanni Schiaparelli (1875). Peters's (1856) formula for  $r$ , the probable error of a single observation, is well known.

In the nineteenth century authors on the general theory included Laplace, Sylvestre Lacroix (1816), Littrow (1833), Adolphe Quetelet (1853), Richard Dedekind (1860), Helmert (1872), Hermann Laurent (1873), Liagre, Didion, and Karl Pearson. Augustus De Morgan and George Boole improved the exposition of the theory.

Andrey Markov introduced the notion of Markov chains (1906) playing an important role in theory of stochastic processes and its applications.

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The modern theory of probability based on the measure theory was developed by Andrey Kolmogorov (1931).

On the geometric side, contributors to The Educational Times were influential.

### **Types of probability:**

There are basically four types of probabilities, each with its limitations. None of these approaches to probability is wrong, but some are more useful or more general than others.

#### Classical Probability:

The classical interpretation owes its name to its early and august pedigree. Championed by Laplace, and found even in the works of Pascal, Bernoulli, Huygens, and Leibniz, it assigns probabilities in the absence of any evidence, or in the presence of symmetrically balanced evidence.

The classical theory of probability applies to equally probable events, such as the outcomes of tossing a coin or throwing dice; such events were known as “equipossible”.

probability = number of favourable equipossibilities / total number of relevant equipossibilities.

#### Logical probability:

Logical theories of probability retain the classical interpretation’s idea that probabilities can be determined a priori by an examination of the space of possibilities.

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### Subjective probability:

A probability derived from an individual's personal judgment about whether a specific outcome is likely to occur. Subjective probabilities contain no formal calculations and only reflect the subject's opinions and past experience.

Subjective probabilities differ from person to person. Because the probability is subjective, it contains a high degree of personal bias. An example of subjective probability could be asking New York Yankees fans, before the baseball season starts, the chances of New York winning the world series. While there is no absolute mathematical proof behind the answer to the example, fans might still reply in actual percentage terms, such as the Yankees having a 25% chance of winning the world series.

In everyday speech, we express our beliefs about likelihoods of events using the same terminology as in probability theory. Often, this has nothing to do with any formal definition of probability, rather it is an intuitive idea guided by our experience, and in some cases statistics.

### Some Of the Examples Of Probability:

X says " Don't buy the avocados here; about half the time, they're rotten". X is expressing his belief about the probability of an event – that an avocado will be rotten – based on his personal experience.

Y says " I am 95% certain the capital of Spain is Barcelona". Here, the belief Y is expressing is only a probability from his point of view, because only he does not know that the capital of Spain is Madrid (from our point of view, the <https://assignbuster.com/the-concept-of-probability-in-mathematics/>

probability is 100%). However, we can still view this as a subjective probability because it expresses a measure of uncertainty. It is as though Y is saying “ in 95% of cases where I feel as sure as I do about this, I turn out to be right”.

Z says “ There is a lower chance of being shot in Omaha than in Detroit”. Z is expressing a belief based (presumably) on statistics.

Dr. A says to Christina, “ There is a 75% chance that you will live.” Dr. A is basing this off of his research.

Probability can also be expressed in vague terms. For example, someone might say it will probably rain tomorrow. This is subjective, but implies that the speaker believes the probability is greater than 50%.

Subjective probabilities have been extensively studied, especially with regards to gambling and securities markets. While this type of probability is important, it is not the subject of this book.

There are two standard approaches to conceptually interpreting probabilities. The first is known as the long run (or the relative frequency approach) and the subjective belief (or confidence approach). In the Frequency Theory of Probability, probability is the limit of the relative frequency with which an event occurs in repeated trials (note that trials must be independent).

Frequentists talk about probabilities only when dealing with experiments that are random and well-defined. The probability of a random event denotes the relative frequency of occurrence of an experiment’s outcome, when

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repeating the experiment. Frequentists consider probability to be the relative frequency “ in the long run” of outcomes.

Physical probabilities, which are also called objective or frequency probabilities, are associated with random physical systems such as roulette wheels, rolling dice and radioactive atoms. In such systems, a given type of event (such as the dice yielding a six) tends to occur at a persistent rate, or ‘ relative frequency’, in a long run of trials. Physical probabilities either explain, or are invoked to explain, these stable frequencies. Thus talk about physical probability makes sense only when dealing with well defined random experiments. The two main kinds of theory of physical probability are frequentist accounts and propensity accounts.

Relative frequencies are always between 0% (the event essentially never happens) and 100% (the event essentially always happens), so in this theory as well, probabilities are between 0% and 100%. According to the Frequency Theory of Probability, what it means to say that “ the probability that A occurs is  $p\%$ ” is that if you repeat the experiment over and over again, independently and under essentially identical conditions, the percentage of the time that A occurs will converge to  $p$ . For example, under the Frequency Theory, to say that the chance that a coin lands heads is 50% means that if you toss the coin over and over again, independently, the ratio of the number of times the coin lands heads to the total number of tosses approaches a limiting value of 50% as the number of tosses grows. Because the ratio of heads to tosses is always between 0% and 100%, when the probability exists it must be between 0% and 100%.



In the Subjective Theory of Probability, probability measures the speaker's "degree of belief" that the event will occur, on a scale of 0% (complete disbelief that the event will happen) to 100% (certainty that the event will happen). According to the Subjective Theory, what it means for me to say that "the probability that A occurs is  $\frac{2}{3}$ " is that I believe that A will happen twice as strongly as I believe that A will not happen. The Subjective Theory is particularly useful in assigning meaning to the probability of events that in principle can occur only once. For example, how might one assign meaning to a statement like "there is a 25% chance of an earthquake on the San Andreas fault with magnitude 8 or larger before 2050?" It is very hard to use either the Theory of Equally Likely Outcomes or the Frequency Theory to make sense of the assertion.

Bayesians, however, assign probabilities to any statement whatsoever, even when no random process is involved. Probability, for a Bayesian, is a way to represent an individual's degree of belief in a statement, given the evidence.

Evidential probability, also called Bayesian probability, can be assigned to any statement whatsoever, even when no random process is involved, as a way to represent its subjective plausibility, or the degree to which the statement is supported by the available evidence. On most accounts, evidential probabilities are considered to be degrees of belief, defined in terms of dispositions to gamble at certain odds. The four main evidential interpretations are the classical interpretation, the subjective interpretation, the epistemic or inductive interpretation, and the logical interpretation.

Theory:

Like other theories, the theory of probability is a representation of probabilistic concepts in formal terms—that is, in terms that can be considered separately from their meaning. These formal terms are manipulated by the rules of mathematics and logic, and any results are then interpreted or translated back into the problem domain.

There have been at least two successful attempts to formalize probability, namely the Kolmogorov formulation and the Cox formulation. In Kolmogorov's formulation, sets are interpreted as events and probability itself as a measure on a class of sets. In Cox's theorem, probability is taken as a primitive and the emphasis is on constructing a consistent assignment of probability values to propositions. In both cases, the laws of probability are the same, except for technical details.

There are other methods for quantifying uncertainty, such as the Dempster-Shafer theory or possibility theory, but those are essentially different and not compatible with the laws of probability as they are usually understood.

Mathematical Treatment:

In mathematics, a probability of an event  $A$  is represented by a real number in the range from 0 to 1 and written as  $P(A)$ ,  $p(A)$  or  $\Pr(A)$ . An impossible event has a probability of 0, and a certain event has a probability of 1. However, the converses are not always true: probability 0 events are not always impossible, nor probability 1 events certain.

The opposite or complement of an event  $A$  is the event (that is, the event of  $A$  not occurring); its probability is given by  $P(\text{not } A) = 1 - P(A)$ . As an

example, the chance of not rolling a six on a six-sided die is  $1 - (\text{chance of rolling a six})$  .

If both the events A and B occur on a single performance of an experiment this is called the intersection or joint probability of A and B, denoted as  $P(A \cap B)$  . If two events, A and B are independent then the joint probability is

For example: if two coins are flipped the chance of both being heads is

If either event A or event B or both events occur on a single performance of an experiment this is called the union of the events A and B denoted as  $P(A \cup B)$  . If two events are mutually exclusive then the probability of either occurring is

For example, the chance of rolling a 1 or 2 on a six-sided die is

If the events are not mutually exclusive then

Conditional probability is the probability of some event A, given the occurrence of some other event B. Conditional probability is written  $P(A|B)$ , and is read “ the probability of A, given B”. It is defined by

If  $P(B) = 0$  then is undefined.

Applications:

Two major applications of probability theory in everyday life are in risk assessment and in trade on commodity markets. Governments typically apply probabilistic methods in environmental regulation where it is called “ pathway analysis”, often measuring well-being using methods that are

stochastic in nature, and choosing projects to undertake based on statistical analyses of their probable effect on the population as a whole.

A good example is the effect of the perceived probability of any widespread Middle East conflict on oil prices – which have ripple effects in the economy as a whole. An assessment by a commodity trader that a war is more likely vs. less likely sends prices up or down, and signals other traders of that opinion. Accordingly, the probabilities are not assessed independently nor necessarily very rationally. The theory of behavioural finance emerged to describe the effect of such groupthink on pricing, on policy, and on peace and conflict.

It can reasonably be said that the discovery of rigorous methods to assess and combine probability assessments has had a profound effect on modern society. Accordingly, it may be of some importance to most citizens to understand how odds and probability assessments are made, and how they contribute to reputations and to decisions, especially in a democracy.

Another significant application of probability theory in everyday life is reliability. Many consumer products, such as automobiles and consumer electronics, utilize reliability theory in the design of the product in order to reduce the probability of failure. The probability of failure may be closely associated with the product's warranty.

Probability Of Winning A Lottery:

Everyone knows that the probability of winning the lottery is a pretty big long shot. How long, however, you probably never really thought about. Your

actual odds of winning the lottery depend on where you play, but single state lotteries usually have odds of about 18 million to 1 while multiple state lotteries have odds as high as 120 million to 1.

If you have ever thought you'd win the lottery, you're not alone. About one out of every three people in the United States think that winning the lottery is the only way to become financially secure in their life. This is a frightening statistic when you sit down and consider what the above odds really mean.

It's time to take a long hard look at the chances of you winning the lottery. While winning the lottery may be something that you want, to show you your chances we'll take a look at a number of remote occurrences that you probably wouldn't like to have happen to you – and probably don't think will ever happen to you – but are still much more likely to happen to you than winning the lottery.

How about the classic odds of being struck by lightning? The actual probability of this happening varies from year to year, but as a good estimate, the National Safety Council says between 70 and 120 people a year die in the US by lightning – so let's take 100 as our base. With the US population being approximately 265 million people, that means that the chances of being killed by lightning are roughly 2, 650, 000 to 1. Not very likely. However you are still 6 to 45 times more likely to die from a lightning strike than you would be to win the lottery.

Now nobody really wants to die from flesh eating bacteria, and with odds at about 1 million to 1, the chances that you will die that way are pretty slim.

Then again, you are 18 to 120 times more likely to die this way than to win the lottery.

What are the chances that if you're playing with a group of four that two of you will get a hole-in-one on the exact same hole? At 17 million to 1, they're better than the chances of you winning the lottery.

What about dying from a snake bite or bee sting? It probably isn't a way that you have imagined that you would leave the earth. You're a whopping 180 to 1, 200 times more likely to die from one of these incidents than win the lottery. That's because the probability of dying from a snake bite or bee sting is about 100, 000 to 1.

Now I know that you are not a bad person and you don't imagine finding yourself on death row for a crime you committed anytime soon. Still, it's a lot more likely that you will be legally executed than win the lottery. In fact, you are 30, 000% to 200, 000% more likely to die in a legal execution than to win the lottery.

If none of the above has convinced you to stop playing the lottery, then I'll bring out my favorite lottery fact. If you drive 10 miles to purchase your lottery ticket, it's three to twenty times more likely for you to be killed in a car accident along the way than to win the jackpot.

Flipping Of Coin:

Coin flipping or coin tossing is the practice of throwing a coin in the air to choose between two alternatives, sometimes to resolve a dispute between two parties. It is a form of sortition which inherently has only two possible

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and equally likely outcomes. Experimental and theoretical analysis of coin tossing has shown that the outcome is predictable.

During coin flipping the coin is tossed into the air such that it rotates end-over-end several times. Either beforehand or when the coin is in the air, an interested party calls “heads” or “tails”, indicating which side of the coin that party is choosing. The other party is assigned the opposite side.

Depending on custom, the coin may be caught, caught and inverted, or allowed to land on the ground. When the coin comes to rest, the toss is complete and the party who called or was assigned the face-up side is declared the winner. If the outcome is unclear the toss is repeated; for example the coin may, very rarely, land on edge, or fall down a drain.

The coin may be any type as long as it has two distinct sides; it need not be a coin as such. Human intuition about conditional probability is often very poor and can give rise to some seemingly surprising observations. For example, if the successive tosses of a coin are recorded as a string of “H” and “T”, then for any trial of tosses, it is twice as likely that the triplet TTH will occur before THT than after it. It is three times as likely that THH will precede HHT.

Are we likely to be struck by lightning?

In the United States, an average of 80 people are killed by lightning each year. Considering being killed by lightning to be our ‘favorable outcome’ (not such a favorable outcome!), the sample space contains the entire population of the United States (about 250 million).

If we assume that all the people in our sample space are equally likely to be killed by lightning (so people who never go outside have the same chance of being killed by lightning as those who stand by flagpoles in large open fields during thunderstorms), the chance of being killed by lightning in the United States is equal to 80/250 million, or a probability of about . 000032%.

Clearly, you are much more likely to die in a car accident than by being struck by lightning.

Probability in Our Lives:

A basic understanding of probability makes it possible to understand everything from batting averages to the weather report or your chances of being struck by lightning! Probability is an important topic in mathematics because the probability of certain events happening – or not happening – can be important to us in the real world.

Weather forecasting:

Suppose a person wants to go on a picnic this afternoon, and the weather report says that the chance of rain is 70%? Will he ever wonder where that 70% came from?

Forecasts like these can be calculated by the people who work for the National Weather Service when they look at all other days in their historical database that have the same weather characteristics (temperature, pressure, humidity, etc.) and determine that on 70% of similar days in the past, it rained.



As we've seen, to find basic probability we divide the number of favorable outcomes by the total number of possible outcomes in our sample space. If we're looking for the chance it will rain, this will be the number of days in our database that it rained divided by the total number of similar days in our database. If our meteorologist has data for 100 days with similar weather conditions (the sample space and therefore the denominator of our fraction), and on 70 of these days it rained (a favorable outcome), the probability of rain on the next similar day is  $70/100$  or 70%.

Since a 50% probability means that an event is as likely to occur as not, 70%, which is greater than 50%, means that it is more likely to rain than not. But what is the probability that it won't rain? Remember that because the favourable outcomes represent all the possible ways that an event can occur, the sum of the various probabilities must equal 1 or 100%, so  $100\% - 70\% = 30\%$ , and the probability that it won't rain is 30%.

#### Bernoulli Trials On Probability:

It happens very often in real life that an event may have only two outcomes that matter. For example, either you pass an exam or you do not pass an exam, either you get the job you applied for or you do not get the job, either your flight is delayed or it departs on time, etc. The probability theory abstraction of all such situations is a Bernoulli trial.

Bernoulli trial is an experiment with only two possible outcomes that have positive probabilities  $p$  and  $q$  such that  $p + q = 1$ . The outcomes are said to be "success" and "failure", and are commonly denoted as "S" and "F" or, say, 1 and 0.

For example, when rolling a die, we may be only interested whether 1 shows up, in which case, naturally,  $P(S) = 1/6$  and  $P(F) = 5/6$ . If, when rolling two dice, we are only interested whether the sum on two dice is 11,  $P(S) = 1/18$ ,  $P(F) = 17/18$ .

The Bernoulli process is a succession of independent Bernoulli trials with the same probability of success.

Uses Of Probability In Our Daily Lives:

I think we use probability routinely in our daily lives. When you get into a car and drive on public roads, we often assume that we have a low probability of being hit by another car. When you pull out onto a busy street crossing 2 lanes of traffic, you judge the speed of the traffic in those lanes. You assume you have a high probability of judging that speed correctly when you cross those lanes. If you did not make that assumption, you probably would not attempt to cross the lanes for fear of being hit by another car.

We assume that we have a low probability of being hit by lightning or a meteor.

When you eat with your hands, you assume your probability of getting sick from germs on your hands is low. Or you wouldn't eat with your hands. You could say the same of eating in a restaurant with reference to food you didn't prepare yourself.

Within assuming many probabilities, I think we'd constantly live in fear of what horrible things might happen to us.

Summary of probabilities:

Event

Probability

A

not A

A or B

A and B

A given B

Other Cases Where Probability Can Be Observed:

You've seen it happen many times—a player in a dice game claims she is “due” for doubles;

strangers discover that they have a mutual acquaintance and think that this must be more

than a chance meeting; a friend plays the lottery obsessively or enters online contests with a

persistent dream of winning. All these behaviors reflect how people perceive probability in

daily life. People who lack an accurate sense of probability are easily drawn in by false claims

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and pseudoscience, are vulnerable to get-rich-quick schemes, and exhibit many of the

behaviors mentioned above.

The modeling and measurement of probabilities are fundamentals of mathematics that

can be applied to the world around us. Every event, every measurement, every game, every

accident, and even the nature of matter itself is understood through probabilistic models, yet

few people have a good grasp of the nature of probability.

Frequentists talk about probabilities only when dealing with experiments that are random and well-defined. The probability of a random event denotes the relative frequency of occurrence of an experiment's outcome, when repeating the experiment. Frequentists consider probability to be the relative frequency "in the long run" of outcomes.[1]

Bayesians, however, assign probabilities to any statement whatsoever, even when no random process is involved. Probability, for a Bayesian, is a way to represent an individual's degree of belief in a statement, or an objective degree of rational belief, given the evidence.

Relation to randomness:

In a deterministic universe, based on Newtonian concepts, there is no probability if all conditions are known. In the case of a roulette wheel, if the force of the hand and the period of that force are known, then the number on which the ball will stop would be a certainty. Of course, this also assumes knowledge of inertia and friction of the wheel, weight, smoothness and roundness of the ball, variations in hand speed during the turning and so forth. A probabilistic description can thus be more useful than Newtonian mechanics for analyzing the pattern of outcomes of repeated rolls of roulette wheel. Physicists face the same situation in kinetic theory of gases, where the system, while deterministic in principle, is so complex (with the number of molecules typically the order of magnitude of Avogadro constant  $6.02 \cdot 10^{23}$ ) that only statistical description of its properties is feasible.

A revolutionary discovery of 20th century physics was the random character of all physical processes that occur at sub-atomic scales and are governed by the laws of quantum mechanics. The wave function itself evolves deterministically as long as no observation is made, but, according to the prevailing Copenhagen interpretation, the randomness caused by the wave function collapsing when an observation is made, is fundamental. This means that probability theory is required to describe nature. Others never came to terms with the loss of determinism. Albert Einstein famously remarked in a letter to Max Born: I am convinced that God does not play dice. Although alternative viewpoints exist, such as that of quantum decoherence being the cause of an apparent random collapse, at present there is a firm consensus among physicists that probability theory is necessary to describe quantum phenomena.