Is complete certainty achievable in mathematics?

Prescribed Title: Mathematicians have the concept of rigorous proof, which leads to knowing something with complete certainty. Consider the extent to which complete certainty might be achievable in mathematics and at least one other area of knowledge.

Mathematics and natural sciences seem as if they are areas of knowledge in which one is most likely to find complete certainty. Many often consider claims that are backed by significant evidence, especially firm scientific evidence to be correct. Once, when I saw my younger sibling snacking on sugar cookies, I told her to limit herself and to try snacking on a healthy alternative like fruit. At first, she shunned my idea, but when I explained to her the numerous health benefits that were linked to eating fruit that was also backed by scientific research, she gave my idea a second thought. Mathematics is heavily interconnected to reasoning and thus many people believe that proofs in mathematics are as certain as us knowing that we are human beings. Both natural sciences and mathematics are backed by numbers and so they seem more certain and precise than say something like ethics. However, things like Collatz conjecture, the axiom of choice, and the Heisenberg uncertainty principle show us that there is much more uncertainty, confusion, and ambiguity in these areas of knowledge than one would expect. For many reasons relating to perception and accuracy, it is difficult to say that one can achieve complete certainty in natural sciences. The same applies to mathematics, beyond the scope of basic math, the rest remains just as uncertain.

A key problem that natural sciences face is perception. Scientific experiments rely heavily on empirical evidence, which by definition depends on perception. In my IB Biology class, I myself have faced problems with reaching conclusions based off of perception. We were once performing a lab in which we had to differentiate between a Siberian husky and an Alaskan malamute, using only visual differences such as fur color, the thickness of the fur, etc. Both animals look strikingly similar and with our untrained eyes we couldn’t correctly identify the differences and so we ended up misidentifying the animals. Perception is also key in cases in which scientists rely on technology like analytical scales to gather data as it possible for one to misread data. Generally speaking, such small nuances usually aren’t significant as scientific experiments are replicated many times. In the grand scope of things, such nuances don’t add up to much as there usually many other uncontrollable factors like confounding variables, experimental factors, etc. Even the state of mind of the researcher or the subject being experimented on can have greater impacts on the results of an experiment compared to slight errors in perception. However, we must note that any factor however big or small will in some way impact a researcher seeking to attain complete certainty. So, natural sciences can be highly precise, but in no way can be completely certain.

In addition, emotions and ethics also play a big role in attaining absolute certainty in the natural sciences. A researcher may write their hypothesis and design an experiment based on their beliefs. The Greek philosopher Ptolemy, who was also a follower of Christianity, came up with the geocentric model, or the idea that the Earth is in the middle of the Universe. Though he may have conducted tons of research and analyzed copious amounts of astronomical calculations, his Christian faith may have ultimately influenced how he interpreted his results and thus what he concluded from them. His conclusions are biased as his results would be tailored to his religious beliefs.

Similar to the natural sciences, achieving complete certainty isn't possible in mathematics. In basic arithmetic, achieving certainty is possible but beyond that, it seems very uncertain. For example, few question the fact that 1+1 = 2 or that 2+2= 4. One can be completely certain that 1+1 is two because two is defined as two ones. The same certainty applies for the latter sum, 2+2 is four because four is defined as two twos. Though certainty seems achievable in basic mathematics, this doesn't apply to all aspects of mathematics. Mathematics makes use of logic, but the validity of a deduction relies on the logic of the argument, not the truth of its parts. For the most part, this truth is simply assumed, but in mathematics this truth is imperative. It could be that a mathematician creates a logical argument but uses a proof that isn't completely certain. Consequently, the mathematician's proof cannot be completely certain even if it may be valid. In other cases, logic can't be used to get an answer. Kurt Gödel's incompleteness theorem states that there are some valid statements that can neither be proven nor disproven in mathematics (Britannica). For, example the incompleteness theorem states that the reliability of Peano arithmetic can neither be proven nor disproven from the Peano axioms (Britannica). Another example would be Goodstein's theorem which shows that a specific iterative procedure can neither be proven nor disproven using Peano axioms (Wolfram). Conclusively, it is impossible for one to find all truths and in the case that one does find the truth, it can't sufficiently be proven. Thus, it is impossible for us to be completely certain.

A major problem faced in mathematics is that the process of verifying a statement or proof is very tedious and requires a copious amount of time. In my theory of knowledge class, we learned about Fermat's last theorem, a math problem that took 300 years to solve. Fermat's last theorem stated that $x^n + y^n = z^n$ has non- zero integer solutions for x, y, z when n> 2 (Mactutor). The problem was first said to be solved by British Mathematician Andrew Wiles in 1993 after 7 years of giving his undivided attention and precious time to the problem (Mactutor). At that time, it was said that the proof that Wiles came up with was the end all be all and that he was correct. However, 3 months after Wiles first went public with this proof, it was found that the proof had a significant error in it, and Wiles subsequently had to go back to the drawing board to once again solve the problem (Mactutor). After another year of grueling mathematical computations, Wiles came up with a revised version of his initial proof and now it is widely accepted as the answer to Fermat's last theorem (Mactutor). But this isn't to say that in some years down the line an error won't be found in the proof, there is just no way for us to be completely certain that this IS the end all be all. Previously, math has heavily reliant on rigorous proof, but now modern math has changed that. In the past, even the largest computations were done by hand, but now computers are used for such computations and are also used to verify our work. The use of computers creates a system of rigorous proof that can overcome the limitations of us humans, but this system stops short of being completely certain as it is subject to the fallacy of circular logic. Due to the many flaws of computers and the many uncertainties about them, it isn't possible for us to rely on computers as a means to achieve complete certainty.

The lack of certainty in mathematics affects other areas of knowledge like the natural sciences as well. For example, my friend is performing a chemistry experiment requiring some mathematical calculations. She isn't very certain about the calculations and so she won't be able to attain complete certainty about that topic in chemistry. As shown, there are limits to attain complete certainty in mathematics as well as the natural sciences. Areas of knowledge are often times intertwined and correlate in some way to one another, making it further challenging to attain complete certainty. Going back to the previous example of my friend, the experiment that she was performing in the areas of knowledge of chemistry also required her to have knowledge in mathematics. Since she was uncertain in mathematics, this resulted in her being uncertain in chemistry as well. A problem that arises from this is that it is impossible for one to determine to what extent uncertainty in one area of knowledge affects one's certainty in another area of knowledge.

On the other hand, it can also be argued that it is possible to achieve complete certainty in mathematics and natural sciences. One can argue that if a science experiment has been replicated many times, then the conclusions derived from it can be considered completely certain. This is also the same in mathematics if a problem has been checked many times, then it can be considered completely certain as it can be proved through a process of rigorous proof. For example, researchers have performed many studies on climate change. From their studies, they have concluded that the global average temperature is indeed rising. This is completely certain as an all researches agree that this is fact as it can be proven with rigorous proof, or in this case scientific evidence.

Though this is a rather compelling argument, we must take some other things into account. If all the researches are completely certain about global warming, are they certain correctly determine the rise in overall temperature? The answer to this question is likely no as there is just too much data to process and too many calculations that need to be done for this. Due to this, the researchers are certain so some degree, but they haven’t achieved complete certainty.

At first glance, both mathematics and the natural sciences seem as if they are two areas of knowledge in which one can easily attain complete certainty. However, upon closer inspection, one can see that there is much more complexity to these areas of knowledge than one would expect and that achieving complete certainty is impossible.

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