Option pricing models

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It is known that most of the option pricing models and techniques employeed by today's analysts are rooted in a model developed by Fischer Black and Myron Scholes in 1973. One basic assumption of BS model is that the stock price is log-normally distributed with constant volatility. However, Fama (1965) and Mandelbrot (1966) found that stock returns exhibit both fat-tailed marginal distributions and volatility clustering. These features are interpreted as evidence of stochastic volatility of financial asset prices. To overcome the shortcoming, many researchers have contributed to substantial new models that incorporate stochastic volatility in the last two decades. It is thus interesting to examine whether the stochastic-volatility option pricing models provide improvements to the BS model.

During the past decade, researchers have begun to study generalized autoregressive conditional heteroskedasticitic (GARCH) models for option pricing due to the superior ability of this class of models to describe asset return dynamics. Duan (1995) developed a theory with respect to which options can be priced when the evolution of the asset return follows the GARCH process. Empirically, Duan (1996), Heston and Nandi (2000), Hsieh and Ritchken (2000), Hardle and Hafner (2000), Duan and Zhang (2001) and Christoffersen and Jacobs (2002) have showed that the GARCH model can be used to capture the pricing behavior of exchange-traded European options. Analytically pricing European options requires the knowledge of the risk-neutral distribution of the cumulative return with respect to a given model. However, the analytical form of the distribution for the time aggregated return is unknown for all GARCH specifications, and thus computing option prices must rely on some time-consuming numerical procedures.

In recent years, researchers have tried to speed up the valuation of European options under GARCH by developing analytical solutions and analytical approximations for specific forms of the GARCH model. Heston and Nandi (2000) developed an analytical formula to price European options when the dynamic of the conditional variance is given by a specific GARCH process. In contrast, Duan, Gauthier and Simonato (1999) developed and analytical approximation for the European option price under GARCH. Their approach utilizes the idea of Jarrow and Rudd (1982) to find an approximate option price under general stochastic process.

Meanwhile, significant computational simplification is achieved when option pricing is approached through the change of numeraire technique. By pricing an asset in terms of another traded asset (the numeraire), this technique reduces the number of sources of risk that need to be accounted for.

Research Objectives

1. To examine the function of the Black-Scholes model and some new models developed from it in modern financial market.

2. To develop the models with less assumptions.

3. To apply the models to the China financial market.

4. To grope different numeraires to simplify the models in China financial market.

5. To compare different option pricing models, such as BS model, GARCH model and so on.

6. To make comment on the models within China financial market.

Literature Review

The Black and Scholes Model:

In order to understand the model itself, we divide it into two parts. The first part, SN(d1), derives the expected benefit from acquiring a stock outright. This is found by multiplying stock price [S] by the change in the call premium with respect to a change in the underlying stock price [N(d1)]. The second part of the model, Ke(-rt)N(d2), gives the present value of paying the exercise price on the expiration day. The fair market value of the call option is then calculated by taking the difference between these two parts.

The GARCH models

There are two types of volatility models: continuous-time stochastic models and discrete-time stochastic generalized autoregressive conditional heteroskedasticity (GARCH) models. On one hand, the continuous-time model can serve as the limit of a certain GARCH model. For example, Nelson(1990a) showed that the GARCH (1, 1) model converged to a certain diffusion model. Duan (1996) argued that most of the existing bivariate diffusion models that had been used to model asset returns and volatility could be represented as limits of a family of GARCH models. As a special case, the particular GARCH option model proposed by Heston and Nandi (2000) was proved to contain Heston's (1993) stochastic volatility models as a continuous-time limit.

On the other hand, the GARCH model has an advantage over the continuous-time model in that the volatility is readily observable in the history of asset prices. As a result, it is possible to price an option only using the information from the observations of asset prices. In contrast, the continuous-time stochastic model has an inherent disadvantage that it assumes that volatility is observable, but it is impossible to exactly filter volatility from discrete observations of spot asset prices in a continuous-time stochastic volatility model.

Consequently, it is impossible to price an option solely on the basis of the history of asset prices. Since volatility is unobservable, one has to use the volatility implied from one option to value other options. Unfortunately, this method is not always feasible especially when the related options are thinly traded. Thus, the GARCH model is chosen over the continuous-time model when comparing the empirical performance of the stochastic option model and the discrete-time model.

The standard Black-Scholes (BS) formula prices a European option on an asset that follows geometric Brownian motion. The asset's uncertainty is the only risk factor in the model. A more general approach developed by Black Merton-Scholes leads to a partial differential equation. The most general method developed so far for the pricing of contingent clains is the martingale approach to arbitrage theory developed by Harrison and Kreps (1981) and others.

Whether one use the PDE or the standard risk-neutral valuation formulas of the martingale method, it is in most cases very hard to obtain analytic pricing formulas. Thus, for many important cases, special formulas (typically modifications of the original BS formula) have been developed.

One of the most typical cases with multiple risk factors occurs when an option involves a choice between two assets with stochastic prices. In this case, it is often of considerable advantage to use a change of numeraire in the pricing of the option.

The basic idea of the numeraire approach can be described as follows. Suppose that an option's price depends on several (say, n) sources of risk. We may then compute the price of the option according to this scheme:

* Pick a security that embodies one of the sources of risk, and choose this security as the numeraire.

* Express all prices in the market, including that of the option, in terms of the chosen numeraire. In other words, perform all the computations in a relative price system.

* Since the numeraire asset in the new price system is riskless (by definition), we have reduced the number of risk factors by one, from n to n-1.

* We thus derive the option price in terms of numeraire. A simple translation from the numeraire back to the local currency will then give the price of the option in monetary terms.