

Quadratic equations



**ASSIGN
BUSTER**

When a ball is thrown up into the air, it makes the shape of a parabola. The equation $S = -5t^2 + v \cdot t + k$ gives the height of the ball at any time. Use the formula to find the height of the ball at any two values of time, t , in seconds.

Given $v = 20\text{m/sec}$ and $k = 100$.

$S = -5t^2 + 20t + 100$ [The height 's' is as a function of time after launching the ball]

At the right hand side of this equation, $(-5t^2)$ is about the effect of gravity - "

All bodies in free fall near the earth surface have the same downward acceleration of $g = 9.8\text{m/s}^2$ " (Beiser, p. 40), the initial velocity $(20t)$ with a positive sign indicates that the ball is tossing upward from the building; and the initial height (100m) where the ball is launched from. (Note that once $t = 0$ at launch, the initial height will be 100m . In other words, we haven't yet launched the ball. We still keep it on top of the 100m high building). For

Formula: $S = -5t^2 + 20t + 100$

Substituting of t with any two values would give us particular values of height:

Let's say that $t = -2$, then:

$$\text{Height} = -5(-2)^2 + 20(-2) + 100 = -5(4) + (-40) + 100 = -20 + 60 = 40.$$

This shows that once $t = -2$, its height will be 40 metres. This is to say that the ball strikes the height of 40 metres on its way down from its turning point. Saying that the time is negative is a bit absurd, isn't it. We would not

say it but the negative sign here is to differentiate the direction of where the ball is heading to at a given point of time.

Let's say that $t = 2$, then:

$$\text{Height} = -5(2)^2 + 20(2) + 100 = -5(4) + 40 + 100 = -20 + 40 + 100 = 120.$$

In this case, the ball strikes the height of 120m on its way up in the air after 2 seconds from tossing it up.

Proof:

We can prove this by finding t using that given height and velocity in the general quadratic formula. Using the height of 120, we end up here: $120 = -5t^2 + 20t + 100$

$$0 = -5t^2 + 20t + -20 \text{ [Subtract 120 from both sides]}$$

Given the General Formula of Quadratic Equation:

We will use 't' in our case instead of 'x'. We know for sure that the discriminant: $b^2 - 4ac$, should not be a negative number because if it does, then we are dealing with imaginary numbers which we need to practice in our 'Calculus'.

As we check our discriminant: $b^2 - 4ac = 20^2 - 4(-5)(-20) = 0$, we know that we are going to have only one root: $t = -20/2(-5) = -20/-10 = 2$. Here we go again, when $t = 2$ seconds, the height will strike the 120metres as mentioned above. As the expression can be factorised: $-5t^2 + 20t + (-20) = (t - 2)(t - 2)$. The answer is 2.