

5-color theorem



Five-Color Theorem The five color theorem, also referred to as the five color map theorem, is a mathematical theorem that was developed from the graph theory. The theorem states that for a given plane divided into adjoining regions, such that it results in a form of a map of countries, no more than five colors are necessary to color the regions of the political map such that no two neighboring areas of the map are colored with the similar color. The five color theorem was developed as an extension to the four color theorem which was proved to have an error in its proof. To understand the five color theory it is necessary to go to the history behind the development of the color theorems. There are three of them, four-color, five-color and six-color theorem. The five color theorem was proved in 1890 showing that five colors suffice to color a map. (Jensen and Toft 61)

It all began with Francis Guthrie. He was a mathematician from British, who in 1852 discovered that he could color the states in the map of Great Britain by means of four colors without coloring of the neighboring countries with the same color. The problem hence arose if it was feasible to color any given map using four colors and it remained an area of interest for a while. The problem was; however, deciphered in 1879 when A. Kempe claimed to have found an explanation to the four color problem and went ahead to publish his solution and proof. In 1890; however, P. Heawood discovered an error in Kempers proof, which led to the demotion of the four color theorem as a credible theory. Heawood was unable to show that there was an error, which could have been colored with not less than five colors, but ultimately proved that Kempe was wrong in his argument. This led to a solution in the color problem with the five color theorem sufficing (Jensen and Toft 61).

In order to proof the five color theorem mathematically, one relates a planar

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graph, G to a certain map. A vertex is placed on every area in the map. Two vertices are then connected with an edge where analogous areas share a boundary in common. This problem is then translated into a graph coloring problem. One is now required to color the graph vertices so that no border has its endpoints with a similar color. This proof relies heavily on the Euler characteristic to illustrate that there, it is mandatory to have a vertex V that is shared by at most five borders. It also relies on the fact that G is a planar. This is to denote that G may be embedded in a plane without necessarily intersecting the borders. Now take out the vertex V , from the planar graph, G . The new graph attained after this will have one vertex less than the original graph G .

At this point, we can presume, by induction that this graph can be painted with just five colors. The vertex V must be linked with five other vertices because if not, it can be painted with the planar graph, G , with a paint not used by the other vertices. Look at the five vertices V_1, V_2, V_3, V_4 , and V_5 that were adjacent to V in cyclical order. It will be found that less than five colors in painting these vertices; therefore, the vertex V can be painted in order to render the planar graph five colored. At this point, we assume that V_1, V_2, V_3, V_4, V_5 are colored with colors 1, 2, 3, 4, and 5 respectively. Now consider the sub graph G_{13} of the reduced G which consists of the vertices colored with colors 1 and 3 only and the points that connect both. If V_1 and V_3 occur in separate linked components, in G_{13} , the coloration can be reversed to that having V_1 . It is then possible to give the color 1 to V hence solving the problem (Jensen and Toft 62).

If on the contrary, v_1 and v_3 are appearing on the same region on G_{13} it is possible to join them. A path is created joining the points that are colored

with color one and color three. Turn to the sub graph G_{24} of the reduced G . This sub graph contains the points that have either color two or color four. Use arguments in the first sub graph G_{13} as above. It is now either realistic to reverse the coloration on a sub graph of G_{24} and paint V with color 2, alternatively we can join the point V_2 with the vertex V_4 through all the points that contain their respective colors. It is; therefore, clear that this path through G_{24} would cut through the path established in the sub graph G_{13} . We can conclude that G can be colored using five colors, which is the complete opposite of the first assumption. This proves the latter, that the map can be colored using at least five colors, which essentially is the five-color theorem (Jensen and Toft 62).

Work Cited

Jensen, Tommy and Toft, Bjarne. Graph Coloring Problems. New York, NY: John Wiley & Sons, 1995. Print.