Pythagorean triplets

We are to investigate the conditions and theory of Pythagorean triplets. Pythagoras' theorem states: in any right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. For numbers to be Pythagorean triplets they have to satisfy the condition:

a2 + b2 = c2

This may be rearranged to give the a2 = c2 – b2 or b2 = c2 – a2, which are useful when calculating one of the shorter sides.

A simple example of this is these numbers: 3 , 4 , 5

Because 32 = 3 * 3 = 9

42 = 4 * 4 = 16

52 = 5 * 5 = 25

32 + 42 = 9 + 16 = 25 = 52

This is the 1st Pythagorean Triple

Another example is: 5 , 12 , 13

Another Example is: 7 , 24 , 25

We can now tell that numbers in the Pythagorean triplets have to be integers and we can now work out the perimeter and area of the triangles.

To work out the perimeter we use the condition: a + b + c = units

1st triplet – 3 + 4 + 5 = 12 units

2nd triplet – 5 + 12 + 13 = 30 units

3rd triplet – 7 + 24 + 25 = 56 units

To work out the area we use the condition: 1/2 * a * b = square units

1st triplet – 1/2 * 3 * 4 = 6 square units

2nd triplet – 1/2 * 5 * 12 = 30 square units

3rd triplet – 1/2 * 7 * 24 = 84 square units

Length of

Shortest side

Length of middle side

Length of longest side

Perimeter

Area

3

4

5

12

6

5

12

13

30

30

7

24

25

56

84

We can now put these results into a table:

To find the lengths of the sides of the 4th and 5th triplets I have to try and recognize a pattern :

Shortest Side

3

` + 2

5

+ 2

7

The difference between the numbers of the shortest side seems to be 2. I can now confidently say that the shortest side in the 4th triple will calculate to be 9.

Middle side

4

+ 8

12 +4

+ 12

24

I have found that the difference (2) of the difference (1) is 4, so,

I can estimate that the length of the middle side for triplet 4 will be 40.

Because 12 + 4 = 16

24 + 16 = 40

Longest Side

Here are the lengths of the longest sides:

Middle Longest

4 +1 5

12 +1 13

24 +1 25

The length of the longest side seems to be the length of the middle side +1

Here are my results for the 4th triple:

Length of Length of Length of Perimeter Area

smallest side middle side longest side units sq units

9 40 41 90 180

I can now see if these numbers satisfy the condition a2 + b2 = c2

This is the 4th Pythagorean triple

I can now put this new result into my table and work out the 5th triple

Length of

Shortest side

Length of middle side

Length of longest side

Perimeter

Area

3

4

5

12

6

5

12

13

30

30

7

24

25

56

84

9

40

41

90

180

11

60

61

132

330

Rules

Smallest side = +2

Middle side = +4 to the difference

Longest side = +1 to middle side

Finding a nth term for the sequences:

Smallest Side

n 1 2 3 4 5

Smallest side 3 5 7 9 11

Difference 2 2 2 2

After studying the grid I have found that the formulae is 2n + 1

Examples : 1 (n) * 2 + 1 = 3

3 (n) * 2 + 1 = 7

5 (n) * 2 + 1 = 11

Middle Side

n 1 2 3 4 5

Middle Side 4 12 24 40 60

Diff 1 8 12 6 20

Diff 2 4 4 4

1/2 2nd Diff 2 2

This sequence is quadratic therefore we know it will include and n2. Now I know the sequences 1/2 2nd difference is 2 I Know the first part to my formulae will be 2n2.

I will now attempt to work out the 2nd part to my formulae by using a table

Triple

2n2

Middle side

Difference between 2n2 and Middle

1

2

4

2

2

8

12

4

3

18

24

6

4

32

40

8

5

50

60

10

I can see now that the difference between n2 and middle side is 2 so I can now say that the formulae will be 2 (n2) + 2.

Examples – 2 (1(n)2) + 2n = 4

– 2 (3(n)2) + 2n = 24

– 2 (5(n)2) + 2n = 60

Thomas Brown G. C. S. E Maths Coursework Mr Tims