

Data each type of  
ball. we can



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DATA ENVELOPMENT ANALYSIS INTRODUCTION: DEA was developed by charnes, cooper and Rhodes using the linear programming method. Earlier it was introduced by Farrell. This method is used to compare the efficiencies of different profitable, non profitable organisations, industry and service departments such as schools, hospitals, manufacturing firm etc. DEA is used to compare: Different firms of a same industry.

Different departments of a same firm. Different branches of a same firm etc.

HOW DOES DEA WORK? The firms or branches which are being compared are called Decision Making Units (DMU).

The best DMU(s) are made as a benchmark and other DMUs are compared with these best DMUs to achieve the best efficiency.  $\text{Efficiency} = \frac{\text{output}}{\text{input}}$ . There may be many output and many inputs.

If there is a single input and single output, then finding the efficiency is quite easy and then comparing them with other DMUs is also an easy task.

However if there are a number of inputs and number of outputs, then finding the efficiency requires assigning some weights to each of the constraints ( inputs and outputs ) and thereby bringing them to a common comparable platform. Eg: Let us suppose that there are managers, engineers and workers working in a branch of a firm manufacturing cricket balls, footballs and basket balls. There may be a number of production units of this firm manufacturing varying number of cricket balls, foot balls and basket balls. They may be different number of managers, engineers and workers. In this case how can we assess the efficiency and how can we compare various branches? The answer is to assign different weights to different constraints

such as salary of each type of manpower and resources required to produce each type of ball. We can take a weighted average of inputs and outputs and compare them with the formula given above.

In this paper I will discuss the step by step method of finding the weights, efficiency and comparison of various DMUs. The basic principle of DEA's working can be summarized as: Comparing the DMUs with a target on the frontier. The frontier is the best practice frontier based on the current set of data available. The input oriented model sees if the DMUs can reduce their inputs to achieve the same output as currently being achieved to meet the frontier.

The output oriented DEA sees if the same inputs can be used to increase the output. One input- one output example: Bank

branch	Transactions/month (output)	No. of Employees (input)	Efficiency (output/ input)
A	1000	85	100.0
B	900	90	94.4
C	1000	110	90.9
D	850	109	77.1
E	900	112	80.4
F	950	108	87.7
G	306	80	38.25
H	750	70	107.1

0No. of Employees (input)1091011121089Efficiency (output/ input)100. 094.

490. 086. 477. 568. 093. 877.

8 The dark black line indicates the frontier. This means that branches A and H lie on the frontier and these act as a bench mark for the other branches. All other branches can follow either of the A and H to become efficient. The ultimate aim is to be as close to the frontier lie as possible.

Each DMU can choose which branch to follow (either A or H) as per their convenience. For example the firm C can choose to be on the horizontal frontier and F can choose to be on the vertical frontier. It is most logical for the branch G to follow the branch A and be on the horizontal frontier. The

slope of the line connecting each point to the origin corresponds to transactions per employee. The highest slope is attained by the line from the origin through A. This line is called efficient frontier.

All points are on or below this line. The name Data Envelopment Analysis is comes from this property because in mathematical terms, such a frontier is said to “envelope” these points. It is assumed that the efficient frontier is effective in the range of interest and is called the “constant returns-to-scale” assumption. In order to know which DMU to follow and keep as a bench mark, draw a line for the origin through the DMUs, and this DMU has to keep the DMU corresponding to that frontier line as its benchmark. (F has to follow H). The ratio of (the length to the DMU from origin) to the (length of the line connecting the frontier to the origin passing through the DMU) gives the DMU efficiency score.

Eg: efficiency score of DMU -E:  $(OE)/(OZ)$ . For an efficient DMU, the efficient score is always 1. Calculation of the target to be obtained: Find the equation of the best frontier line. (Slope from the line from origin to the DMU point. Coordinates of each DMU are known). Find the equation of the line through the DMU under consideration (similar procedure). Find the intersection of both the lines.

This gives the input and output to be obtained. (Either increase the output or decrease the inputs to match the target DMU). This single input and single output can easily be analysed using a graphical representation. But as the number of inputs and outputs increase, it will be very difficult to analyse through a graphical representation. Therefore we use linear programming for

such higher end problems. Two input –one output example: Let there be 5 firms A, B, C, D, E producing plastic water bottles. The inputs are labour and capital invested. The output is the number of plastic bottles produced per month.

Firm	output	labour	capital
A	30	28	25
B	25	20	38
C	43	54	6
D	11	10	8
E	7	12	5

$7.509.335.005.006.33$  output/ labour

$2.732.803.132.863.17$  output/ capital

Let  $\beta_1 = 0.63$   $\beta_2 = 0.27$ ,  $\beta_1 + \beta_2 = 1$ . The productivity w. r.

t labour is output/labour and is as shown in the table. The productivity w. r.

t. capital is output/(capital ) and is as shown in the table. For calculating the productivity w. r. t both labour and capital, we need to Assign some weights to each of the inputs ( $\beta_1$  and  $\beta_2$ ).

Total productivity factor (TPF) is defined as:  $output / ((X_1^{\beta_1}) * (X_2^{\beta_2}))$ ; j – the firm A, B.. E. The TPF should lie between 0 and 1.

Solving for the weights using the LP model (variable weights method):

Maximize  $(\sum_{i=1}^n v_i y_{iq}) / (\sum_{j=1}^m u_j x_{jq})$  Sub to  $(\sum_{i=1}^n v_i y_{iq}) / (\sum_{j=1}^m u_j x_{jq}) \geq 1$  ;  $k= 1, 2, \dots, n$   $u_i \geq 0$   $i= 1, 2, \dots, m$   $v_j \geq 0$   $j= 1, 2, \dots, n$ . This above model can also

be written in form of a matrix form which is called CCR (Charnes, Cooper and Rhodes method). Let the weights of output be v and the weights of inputs are  $u_1$  and  $u_2$  respectively. Then the effective efficiencies of all the firms can

be written as: OUTPUT (WT v) LABOUR (WT  $u_1$ ) CAPITAL (WT  $u_2$ ) EFFECTIVE EFFICIENCY  $A \frac{30v}{4u_1+11u_2}$   $B \frac{28v}{3u_1+10u_2}$   $C \frac{25v}{5u_1+8u_2}$   $D \frac{20v}{4u_1+7u_2}$   $E \frac{38v}{6u_1+12u_2}$  The numerator or the denominator can be modified to achieve maximum effectiveness.

If the numerator is increased keeping the denominator constant, then we are increasing the output keeping the input constant. If we decrease the denominator keeping the numerator constant, it means that we are decreasing the inputs keeping the outputs constant. Effective efficiency for A can be written as:  $\frac{30v}{4u_1+11u_2} \geq 1$ . This can be re written as  $30v - 4u_1 - 11u_2 \geq 0$  LP model ( maximizing the output keeping the input constant): For firm A: Max :  $30v$  ———equation 1 Sub to :  $4u_1 + 11u_2 = 1$  ———equation 2  $30v - 4u_1 - 11u_2 \geq 0$ ; For firm A ———equation 3  $28v - 3u_1 - 10u_2 \geq 0$  ; For firm B ———equation 4  $25v - 5u_1 - 8u_2 \geq 0$  ; For firm C ———equation 5  $20v - 4u_1 - 7u_2 \geq 0$  ; For firm D ———equation 6  $38v - 6u_1 - 12u_2 \geq 0$  ; For firm E ———equation 7  $v, u_1, u_2 \geq 0$  . For firm B Max :  $28v$  ———equation 8 Sub to :  $3u_1 + 10u_2 = 1$  ———equation 9  $30v - 4u_1 - 11u_2 \geq 0$ ; For firm A ———equation 10  $28v - 3u_1 - 10u_2 \geq 0$  ; For firm B ———equation 11  $25v - 5u_1 - 8u_2 \geq 0$  ; For firm C ———equation 12  $20v - 4u_1 - 7u_2 \geq 0$  ; For firm D ———equation 13  $38v - 6u_1 - 12u_2 \geq 0$  ; For firm E ———equation 14  $v, u_1, u_2 \geq 0$  . Similarly for other firms only the equations 1 and 2 change and all others remain the same. Note that the equation of constraints of that particular firm also have to be considered in finding the LP model because, each firm ( including itself for the basis for the DEA) Given the equations for the LP model, we can solve for the weights using the Excel.

By writing the dual of the model above, we get can reduce the outputs keeping the inputs constant. The weights will vary with the firm. Each firm has a different set of weights. For Firm A, the weights are calculated using the LP model as per the equations above. Similarly the weights of all other firms may also be calculated. VU1U20. 0311690. 0571430.

07013FirmFirm no. OUTPUTLABOURCAPITAL A130411MAX0.

935065B228310C32558LHSSIGNRHSD420471= 1E538612-0. 06494 <= 00

<= 0-0. 06753 <= 0-0. 0961 <= 00 <= 00 <= 0Dual of the model for firm

A: Let  $\theta$  be the effectiveness and  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$  be the weights of the dual function.

Min  $\theta$ , Sub to  $30\theta_1 + 28\theta_2 + 25\theta_3 + 20\theta_4 + 38\theta_5 \leq 30$   $4\theta_1 - 4\theta_2 - 3\theta_3 - 4\theta_4 - 6\theta_5 \leq 0$   $11\theta_1 - 11\theta_2 - 10\theta_3 - 8\theta_4 - 7\theta_5 \leq 0$   $1\theta_1 + 2\theta_2 + 3\theta_3 + 4\theta_4 + 5\theta_5 \leq 0$

— free On solving for this dual model using Excel, we get the weights of the  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$  and the effectiveness also.  $\theta_j, j= 1$  to corresponds to

firm A, B, C, D and E respectively. The non-zero weights of the firm indicate that they are the benchmark firms. The other firms may follow them by decreasing the inputs. The Dual indicates that the effectiveness can be achieved by changing the denominator or the input values. The LP model for firm A is shown below. This means that the firms B and E are the benchmark DMUs. The values of  $\theta_2 = 0$ .

702 indicate that the firm A has to follow firm B 0. 702 times to achieve the maximum effectiveness. Firm A can follow firm E also. It has to follow  $\theta_5 = 0$ . 272 times firm E to get maximum effectiveness.  $\theta_1, \theta_2, \theta_3, \theta_4 \leq 50$ .

93506500. 701299000. 272727lhssignrhs302825203830  $\geq 304-4-3-5-4-6-2$ .

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$2E-16 \geq 0$  11-11-10-8-7-12-1.  $3E-15 \geq 0$  The LP model of the dual of firm B is shown below.  $1? 2? 3? 4? 5$  10100-4.  $7E-13$  lhs sign rhs 302825203828  $\geq 283-4-3-5-4-69$ .  $28E-13 \geq 0$  10-11-10-8-7-12-6.

$3E-13 \geq 0$  We can see that the only  $? 2$  is non zero at 1 and the rest are zeros. This means that firm B has to follow itself to achieve the max effectiveness. The value of  $? = 1$  indicates that the effectiveness has reached the maximum value. The solution of Firm E is shown below.

Observe that now only  $? 5$  is non zero at 1 and the rest are zeros. This means that the firm has already reached the maximum effectiveness and need not change again. The best firm may also try to improve its own performance.

But this method doesn't deal with that. It only forms a benchmark for the slightly non performing firms. Observe that the effectiveness of the non performing firms is not equal to unity even with the variable weights.  $? A = 0$ .

$935$  and not 1.  $1? 2? 3? 4? 5$  101.  $9E-12$  001 lhs sign rhs 302825203838  $\geq 386-4-3-5-4-62$ .  $51E-12 \geq 0$  12-11-10-8-7-12-2.  $6E-12 \geq 0$  Two input - three output example: Consider four universities where the inputs are funds received from public and fees, and professors available. The outputs maybe the number of students, average salary package of the graduates passed from the university and the number of academic publications per year.

university Input-1 (income in crores) Input-2 ( number of professors) Output-1 (students in 1000) Output-2 (average salary in rupees, 1000s) Output-3 (publications in



1000s)A100250055012B8022004408C6015003556D10027504408Fixed

weights : This method is called constant return to scale (CRS) method. Let

the weights of input 1=  $u_1$ , input 2=  $u_2$ , output1=  $v_1$ , output2=  $v_2$ ,

output3=  $v_3$ . The efficiency is given by :  $v_1 \cdot$

$output_1 + v_2 \cdot output_2 + v_3 \cdot output_3$   $u_1 \cdot input_1 + u_2 \cdot input_2$  As we can see this is

a weighted average of the inputs and the outputs. As given by a fixed

weights. For instance let  $u_1 = 5$ ,  $u_2 = 3$  and  $v_1 = 4$ ,  $v_2 = 5$ ,  $v_3 = 7$ , then the

efficiency is given as:  $\frac{ip-1 \cdot ip-2 \cdot op-1 \cdot op-2 \cdot op-3}{Efficiency (\%)}$  A1002500550124.

425B80220044083. 886C60150035566. 854D100275044083.

109WEIGHTS 53457As we can see the best university is university-C and the

worst is the university-D. We have taken the weighted average to find out

the weights. I have taken these weights as an arbitrary numbers but they

can be realistic as per the situation. Limitations of fixed weights: The

justification of the weights assigned to the different constraints is

debatable. The change in efficiency ratings due to these weights are also

questionable. Linear regression methodology allows some of the points to lie

above the frontier line also. Therefore it violates the basic assumption that

all values must lie below a benchmarking line and it is not possible to create

a benchmark in the first place.

The universities (as in this case) maybe situated in various different locations

where the standard fee structure and standard salary of the graduates are

different. Therefore assigning common weights is not recommendable.

Variable weights: This method was developed by Banker, Chranes and

Copper in the year 1984.

This method is more acceptable method as it considers the variable weights to each of the different DMU. We use the LP programming method to find the weights to each of the DMU which is based on all the DMUs. It is we form an LP equation based on the equations of all the variables. For university A: The efficiency is given by  $???\ = (5*v1) + (50*v2) + (12*v3) / (100*u1) + (2500*u2)$  This efficiency has to be less than or equal to 1. Similarly we can write the efficiencies for all other DMUs.

These equations can be used to find the efficiencies using