De-icing operations at minnesota saint paul

EXECUTIVE SUMMARY

This report explains the results of research into how de-icing operations at Minnesota Saint Paul could be improved. The findings were that this airport should use a combination of infrared heating and antifreeze spraying in order to deice aircraft in a fast, safe, cost effective and environmentally friendly way. The de-icing process is called the " drive through method" and this report has backed up its investigation through detailed calculations and the use of a decision matrix to compare the benefits of the drive through method against using others. Although there are some problems such as its relatively long payback time of 2. 5 years, plus risk the airport may have to pay licence fees due to patents on the technology, the benefits of the drive through method outweigh these drawbacks. One reason for this is that after the payback period the airport will make an annual saving of approximately $7, 080, 000

INTRODUCTION

Rationale for the research

The process of removing ice, frost or snow from the surface of an aircraft is known as aircraft de-icing. This is an essential procedure because if these substances accumulate on an airplane they will amplify the drag force that the plane experiences. This will reduce the ability of its wings to produce enough lift force to allow it to take-off or manoeuvre whilst in flight. Also, damage could be caused if a large piece of ice dislodges from the plane and hits sensitive components like its engine. This could lead to passengers losing their lives in a crash and others losing their properties. The Federal

Aviation Administration, FAA, regulate all major civil aviation operations in America. One of their rules is that aircraft must be free of ice before takeoff and during flight.

Background

This report uses Minneapolis Saint Paul (MSP) airport, Minnesota, as its primary case study. This is because aircraft at this airport frequently needs to undergo deciding as a result of the cold climate before, during and after winter in that region. MSP airport spans 3, 400 acres, has five runways, five de-icing pads and " served more than 32 million travellers in 2009 making it 15th in the United States and 30th in the world in terms of number of passengers served annually." (MSP Airport, 2010)

De-icing is currently carried out at MSP airport by operators who spray an ethylene-glycol based aircraft de-icing fluid onto planes. Storm water drains to collect the waste fluid, before it is transported by a truck to a recycling facility to be treated. It needs treatment because its high Biochemical Oxygen Demand (BOD) makes it harmful to the environment.

Research Aims and Objectives

This investigation aims to find a method and chemical to use for de-icing planes which is more environmentally friendly, cheaper and faster than that which is currently used in MSP airport.

It should be a " total solution technology" which eliminates/reduces all the problems that the current de-icing method causes without producing detrimental side effects.

These aims will be achieved by completing the following objectives:

Identify a substitute de-icing fluid which has the same/better de-icing functionality but costs less and is safer than the one currently used in MSP airport.

Find a better way to recycle the waste de-icing fluid

Ensure that the new substitute chemical allow de-icing fluid to be recycled

Find another way to remove snow from aircraft

GENERIC DESIGN PROCESS

Organisations often use a generic design process as they turn a product idea into a manufactured item. Using a systematic, well organised designed process helps to reduce the research and development time that a novel product experiences. The design team for this project used a generic design process and the actions that they took at each stage are detailed below:

Product Planning - First the team used product planning to help make that reliable and valid research was carried out straight away. This started this by withholding a discussion in which they clarified the aim and objectives of the project. Then they identified their strengths and weaknesses in relation to these objectives to help them choose which duties they were responsible for. Finally they agreed upon timescales in which to achieve each objective.

Identification of Customer Needs - Customers needs guided the team's product innovations that were found. The team held interviews with a representative from each major stakeholder group in the airline industry, such as the airline manager and spray operator, to allow them to voice their needs. This made it easier for the team to set product specifications and design a product that they would approve.

Establishment of Product Specifications - The customer needs were ranked in order of their importance. The rank of each need was proportional to a weighting, w, which was later used in a decision matrix. This information was used to generate product specifications which were further defined using metrics.

Generation of product ideas - A brainstorm was held to generate product ideas. This was useful because it encouraged the team to build ideas on top of one another. From this they saw similarities between ideas and linked some of them to define a total solution technology.

Selection of product ideas - A decision matrix was used to quantitatively compare the importance of each customer need in relation to the product ideas. The product with the highest score was selected for testing.

Testing - The total solution technology was further evaluated in terms of its performance and economic viability. Because it was very beneficial its specifications were was sent to manufactures so that they could build a prototype.

Figure 1 - An illustration of the generic design process

Figure 1 illustrates the product design methodology which begins with product planning and ends with testing and manufacture. The dotted lines show that if one stage of the design process did not give advantageous results the team would go one or multiple stages back in order to refine their previous intentions. Then they would advance through each stage of the process again until they reached the final testing and manufacturing stage. This procedure of assessing and re-assessing product developments ensured that poor designs were eliminated or improved before they reached the testing and manufactures stage.

NEEDS

Various customers have an interest in aircraft de-icing operations; these individuals are referred to as stakeholders. The success of this venture will depend on how well it meets the needs of these individuals. Hence, the team evaluated each stakeholder's need using an interview and ranked it according to its importance.

Information Gathering

The stakeholders were identified on the basis on who will pay for, sell, use and operate the de-icing technology, these included airline pilots and passengers etc. They were interviewed and their responses are summarised below.

Spray operator

Q1: How do you deice a plane?

" A container on a truck is filled with de-icing fluid which is mixed with water to a 50% concentration by volume. I sit in an enclosed cabin and heat the fluid onboard the truck to 70oC before I spray it onto the plane until all the ice melts."

Airport manager

Q2: How important is the BOD of a de-icing fluid?

" Very important, we pay the treatment works about $0. 35 per US gallon and the price goes up if the BOD increases. Our airport is fined if the BOD5 discharged to the environment exceeds 900 tonnes per year.

Q3: How long would you expect to wait for a return on your investment?

" 1. 5-2 years"

Q4: How much does ethylene glycol cost?

" The average is $5-7 per gallon."

Q5: What precautions were taken since your last incident?

" We thought of retraining staff, but now operations are run by an outsourced business.

De-icing pads and a drainage system were installed."

Aircraft manager

Q6: What do you think of integrating a heating system onto aircraft?

" Not ideal for commercial planes because it's expensive"

Air traffic controller

Q7: How severe is the disruption caused by aircraft de-icing?

" During the peak of the winter season there are regular delays. Aircraft must be de-iced again if they exceed a holdover time of 5 minutes."

Ranking of needs and 'sanity check'

The team identified the most important stakeholder needs and ranked them as listed below.

Each need was given a weighting, w, according to its rank as part of a 'sanity check'.

Table 1 - The rank of each stakeholder need and its weighting

Need

Weight-ing, w

Reasons

1) Safety

4

Safety was the top concern of every stakeholder.

Aircraft at MSP airport can only carry passengers if they obey safety regulations set by the FAA.

2) Lifetime

Cost

3

Some stakeholders disagreed on financial issues regarding how much cash should be spent on certain items. e. g most spray operators would like luxury de-icing cabins whereas airport managers would rather invest the money.

However, all stakeholders agreed that no de-icer with a high lifetime cost would be acceptable.

3) Speed

2

Slow de-icing can cause profit losses due to delayed flights.

4)

Environmental impact

1

Although the airport will be fined for causing excessive pollution these costs are normally absorbed by customers.

ESTABLISHMENT OF PRODUCT SPECIFICATIONS

Most of the needs highlighted by stakeholders were expressed in a qualitative manner. They needed to be converted into specifications in order to avoid trivial yet expensive improvements being made to MSP airport. To do this the worst case de-icing conditions that could potentially take place at MSP airport were defined and as part of sanity check. Next metrics were used establish specifications for product and process design methods that could cope with the worst case de-icing scenarios at the airport.

Specification 1 - Annual length of operation

The dates in which the planes will need to be de-iced ranges from November to early April because on these dates the temperature in Minneapolis Saint Paul is below freezing.

Therefore, any new de-icing method must be able to operate through this period of 5 months and 1 week (157 days) every year. (Figure 2)

Figure 2- The average lowest temperature in MSP every year from 1971-2000

Specifications 2 - The case study aircraft and its ice coverage

The mass of ice on a plane was calculated using a worst case scenario, which was that one of the biggest commercial passenger planes; the Boeing-747 (BBC, 2007) needed to be de-iced.

It was assumed that the top area of both its wings was covered by a 1cm thick layer of ice.

Wing area (Boeing-747) = 541. 2 m2 (Airliners, 2010)

Ice Thickness = 0. 01m

Ice Coverage 100%

 œ Volume of ice on wings, Volice = 5. 412 m3 (App. 1, Eqn 1)

Density of ice, Ï ice = 917kg/m3 (Kotz, 2009)

 œ Mass of ice, = 4962. 8 kg (App. 1, Eqn 2)

Specifications 3- Heating Duty

Latent heat of fusion of ice= 333 kJ/Kg (Bird, 2003)

 œ Minimum heating duty = 1652. 6 MJ (App 1, Eqn 3)

Specifications 4 - The substitute de-icing fluid

The atmospheric temperature in MSP airport during the de-icing season is 20oF (-29oC) therefore the substitute ADF freezing point must be less than 20oC in order to maintain its functionality. Additionally it must provide a freezing point depression of more than 20oC as a result of this ambient temperature. All the stakeholders agreed that product safety is of upmost importance, so the substitute ADF should be less toxic than ethylene-glycol. Additionally they agree that the annual raw material costs of the new dicing method should be less than that of the current method. Two ways to do this are to use less dicing fluid per plane in the first instance; this should be less than the 408 gallons per aircraft required by the current de-icing method (App. 1, Eqn. 9), or to recycle the de-icing fluid.

GENERATION AND SCREENING OF IDEAS

The team worked individually and brainstormed together to help maximise the number of good ideas that were generated. Whilst doing so they reflected upon the product specifications to and stakeholder needs to help screen ideas.

De-icing Ideas

De-icing boots - Rubber boots are attached to the front edges of wings on the plane. The aircraft inflates these boots with air to cause ice that remove ice that has accumulated on them. An unacceptable risk of using this method in MSP is that the system must be activated as soon as a before an large ice layer can form and hit other parts of the plane when it dislodges.

Bleed air- In this method hot air from the aircraft engines is blasted on to the ice to melt it. Although this could melt ice very quickly the airport manager at MSP airports commented that integrating heating systems on to aircraft is " not ideal for commercial planes because it's expensive".

Mechanical Scraping/Blowing

Employees use brushes, or cloths to physically push ice off the aircraft. This method would be very easy to put into practice at MSP because the equipment the required is very cheap. But it is more likely that damage will be done to airplanes as the employees scrape ice so the resulting aircraft maintenance cost is could be very high.

A propylene glycol based de-icing fluid - Propylene glycol is a popular de-icing fluid and is regarded as non-toxic, hence it was chosen for further investigation.

Decision Matrix

The interviews that were conducted with the stakeholders highlighted that some customer needs are more important than others. Although the importance of some specific needs differed in each stakeholder group four needs were consistently rated as essential.

These criteria were analysed in a decision matrix with weighing marks taken from Table 1

Table 2- Decision Matrix which focuses on the alternative deinking methods

ALTERNATIVES

Ethylene Glycol

(Benchmark)

Propylene Glycol

Infrared + Propylene-glycol

Criteria

Weighting

w

Rating

Score

Rating

Score

Rating

Score

Safety

5

2

10

4

20

7

35

Lifetime Cost

4

5

20

5

20

6

24

Speed

3

5

15

4

16

6

18

Environmental impact

2

1

2

5

10

6

14

Total

14

13

47

18

66

25

91

According to this decision matrix the best de-icing solution should be based on a combination of infrared and propylene glycol de-icers is the best as this alternative has the highest sore. The second best method would be to use an ADF which is based on propylene glycol rather than ethylene glycol.

SELECTION OF IDEAS

A comparison of ethylene-glycol and propylene glycol based de-icing fluids

The most freezing point depressants in aircraft de-icing fluids in the US are ethylene glycol (EG) and propylene glycol (PG). Because PG and EG have a similar lifetime cost, in this chapter the chemicals will be compared at a 50% concentration by volume in terms of their safety, de-icing speed and environmental impact because these are three of the most important needs the stakeholders.

Safety

Ethylene glycol has a relatively high toxicity when compared to Propylene-Glycol. It has been classed by the US congress as a hazardous air pollutant (HAP), if 2268 kg or more escapes into the environment within 24 hours users are obligated to report the event under the Comprehensive Environmental Response Compensation and Liability Act (CERCL). However propylene glycol is not classified as a HAP, and users are not required to inform CERCL if it is released.

Both chemicals are said to be non-lethal if humans if they are breathed in with air or adsorbed through skin. However, unlike PG, EG is toxic to humans and mammals if it is ingested directly. Even though PG it is regarded as non-toxic it is still a health hazard because it uses oxygen as it biodegrades which could cause some organisms to suffocate.

De-icing Speed

The freezing point of Propylene-glycol is -34oC which is slightly higher than ethylene-glycol which freezes at -37oC. This is one of the reasons why MSP airport still uses EG. Another is that a lower volume of antifreeze is required for EG to achieve the same freezing point depression as PG.

MSP airport suffers severe weather extremes so it needs to use an ADF which is reliable, especially in extremely cold weather as this is generally when it takes longer to deice aircraft

Propylene-glycol's boiling point is 106oC whereas ethylene glycol has a boiling point of 102. 2oC for ethylene glycol. In this case Propylene-glycol is the better choice because it can transfer heat to ice at a higher temperature than ethylene glycol.

Discussion

There are some drawbacks with regard to using ethylene glycol as a de-icing fluid, especially concerning its environmental impact. But, overall EG and PG have a similar de-icing performance level.

The Rate of Melting due to Heat Conduction Alone

The ADF is heated to 70oC before it is sprayed onto the plane, this heat alone will cause the ice to melt through heat conduction.

The rate of this heating was calculated using equation 1a,

1a)

Where Q= is the heat flow rate in the X-direction in kJ/s, A is the area normal to the direction of heat flow in m2, dT/dx is the temperature gradient and K is the thermal conductivity of ice

The thermal conductivity of ice at -20oC is 2. 39 W/mK, the area normal to the direction of heat flow is equal to the wing area of the Boeing 747 = 541. 2m2, the temperature change that occurs is (70oC- -20oc)= 90oC and the thickness of the ice x is 0. 01m.

Hence the rate of heat transfer by the temperature of the de-icing fluid alone is

2a)

The sensible heat is kJ (App. 1, Eqn 12)

The latent heat of fusion is 1652612. 4 kJ, (App. 1, Eqn 3)

Summing the above gives the amount of het required to melt the ice on a Boeing 747 from a starting temperature of -20oC which is

Hence minimum time that it would take to melt the ice on the plane by heat conduction alone is

This is a very fast time, especially as according to the project brief, de-icing at MSP airport normally takes 10 minutes per plane. One reason reason for this difference is due to the fact that the ADF fluid is not always in full contact with ice, only its bottom surface is. Additionally these calculations assume that heat transfer occurs over the whole of each wing evenly, which is not the case in real life because de-icing fluid is sprayed onto the wing in different locations systematically. A final cause of this difference could be due to the fact that snow has a lower heat conductivity, k, than ice and so any snow on a plane would lower the rate of heat transfer from the ADF.

TOTAL SOLUTION TECHNOLOGY

Drive-Through De-icing

Figure 3 - An illustration of the drive through de-icing technology

The final design concept was to conduct de-icing operations as a drive through system.

First the aircraft enters the Infrared hangar where and warm air blows snow of its wings whilst they are heated by infrared radiation for a typical duration of 5-10 minutes. Ice on the plane melts onto an inclined slope and the wastewater is channelled into the waste collection zone.

Water is channelled through existing storm water pipes into infrastructure to the located under the floor of the spraying and waste collection area. The wastewater is pumped out and transferred by a truck into the airport's detention ponds.

When the wastewater has been removed compression plugs are removed and the plane moves into the spraying area. Here it is sprayed with propylene glycol for up to 5 minutes to help prevent any ice forming on it before takeoff.

The propylene glycol drains into a separate area of the collection chamber and is pumped to a detention pond until it is due to be recycled. Finally the compression plus are reinstalled so the system is ready to deice another plane.

Recycling Solution

Minneapolis currently uses three Glycol Recovery Vehicles to collect wastewater. Using the IR facility eliminates the need to deice aircraft using glycol by around 90%, so no more of these vehicles will need to be bought if the new technology is adopted. Also the airport has saved capital costs by using their existing storm water drains to collect both glycol and storm water. These storm drains can and should continue to be used if MSP airport adopts the Drive-Through De-icing system to save cash and time during the installation of the new system. According to (Big book) " Careful management of the retention systems enables the airport to collect enough wastewater with high glycol concentrations to make glycol recycling/recovery economically viable." The majority of glycol which is recycled is sold to manufacturers who use it in other glycol-based products.

ANALYSIS OF ECONOMIC VIABLITY

Rate of return on investment (ROI)

3a)

Payback Time

3b)

= 2. 5 years

Unfortunately the payback time on the drive through de-icing method is not fast enough to satisfy the Airport Managers at MSP who expect a payback time of " 1. 5-2 years". A higher rate of return on investment would reduce the payback time so it would be wise to look into additional ways to reduce the cost of de-icing using this technology, and ways to improve its efficiency.

Patent issues

Because this total solution technology uses de-icing methods which have been used in industry previously, but it combines them in a unique way it is difficult to assess whether or not it can be patented. Moreover, it is expected that it would take a long time to patent the product even if it were possible because of its complexity and use of old de-icing ideas. The author has recommended that MSP airport consults a lawyer regarding these matters if they do not need to use the de-icing solution immediately. If MSP airport do need to use the technology immediately they might have to pay license fees to one or more patent owners.

CONCLUSION

The findings from this investigation have shown that the solution technology that MSP airport should use to improve its aircraft de-icing solution involves using infrared heating and a propylene glycol as a substitute chemical for ethylene-glycol. The process is called the " drive through method" and the major advantages of this hybrid solution are that it meets the needs of its stakeholders by being safe, having a low lifetime cost, fast aircraft de-icing rate and low environmental impact. Evidence of this has been provided through a decision matrix and several mathematical evaluations.

Unfortunately this solution has a payback time of 2. 5 years, so research should be carried out to reveal how to make the annual rate of return on the investment higher. Finally, this hybrid system uses patented technology so MSP airport might have to pay licence fees for a number of years if it used the drive through method. However, after the payback period the annual saving of $70, 844, 300 per year outweighs any of these drawbacks.

Further Recommendations

Find ways to cut the operating cost of the Drive-Through De-icing system as this will lift the annual rate of return that this technology provides. If the rate of return is high enough the payback time will drop below 2 years and the airport managers in MSP will have this need fulfilled.

Use hot air to blow snow off the aircraft in the IR hangar. This will help to melt the snow and ice too, however the cost of warmin+

3. 0g air may offset the benefit of a faster de-icing time. Nb, as shown by the weightings, w, in Table 1, the lifetime cost of the technology is more important that the de-icing speed that it provides.

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APPENDICIES

Appendix 1

Specifications 2 -The case study aircraft and its ice coverage

The mass of ice on a plane was calculated using a worst case scenario, which was that one of the biggest commercial passenger planes; the Boeing-747 (BBC, 2007) needed to be de-iced. It was assumed that the top area of both its wings was covered by a 1cm thick layer of ice.

Wing area (Boeing-747) = 541. 2 m2

Ice Thickness = 0. 01m

Ice Coverage 100% of wing area

(1)

Density of ice, Ï ice = 917kg/m3 (Kotz, 2009)

(2)

Specifications 3 -Heating Duty

The ice on the aircraft needed to be provided with enough energy to overcome its 333kJ/Kglatent heat of fusion, L, to melt.

The amount of heat energy, Q, required to achieve this was calculated using Equation 3

(3)

(Bird, 2003)

Specifications 4 - The substitute de-icing fluid

In this section the volume of ethylene glycol and propylene glycol needed to deice a single Boeing-747 will be calculated and compared. The metrics are based upon a 50% by volume solution of each chemical.

Volume of Ethylene Glycol Required

The freezing point depression of an ideal solution is given by the formula

(5)

Where âˆ†Tf represents the freezing point depression, m, is the solute concentration and

Kf represents the freezing point depression constant of water which is 1. 86°K·kg/mol (Kilter P., Mosher M. and Scott A. Andrew scott, 2008)

The desired freezing point depression, âˆ†Tf is 20oC because winter temperatures in MSP fall to that temperature, according to the project brief.

(6)

The number of moles of glycol required to achieve the freezing point depression:

(7)

The chemical formula of ethylene-glycol is CH2OHCH2OH

Mr Carbon = 12, Oxygen= 16, Hydrogen= 1

 œMr ethylene glycol = 2*16+2*12 + 1*6= 62 g/mol

Hence the mass of ethylene-glycol required per plane is:

(8)

The density of UCAR ADF at 20oC is 8. 9 lb/gal (US) (1. 07 kg/L) (SAE AMS 1424 Ethylene Glycol-Based Type I Fluids page 8)

Hence the volume of ethylene-glycol required is at least 408 US gallons

(9)

Because the de-icing fluid used in MSP airport is 50% ethylene-glycol and about 50% water, de-icing a single plane would need double the amount of ADF which works out at 6, 184 litres.

The cost of ethylene glycol per aircraft

In general the chemical components in de-icing fluids, such as water, have a total cost which is substantially less than that of ethylene-glycol. Hence the cost of these components in de-icing fluid solutions as deemed insignificant and ignored.

According to the airport manager that was interviewed the minimum cost of a gallon of ethylene glycol is $5 per US gallon

(10)

In MSP airport there is an average of 293 takeoffs per day from 5 de-icing pads (MSP Airport, 2010)

Assuming that during the de-icing season in MSP lasts 157 days and every plane needs to be de-iced the annual cost of ADF is approximately

(11)

Sensible heat required

According to the project brief winter temperatures in Minneapolis Saint Paul fall to-20oC. Ice must be brought to its melting point of 0oC before it can undergo a phase change from solid to liquid.

Where Cp is the specific heat capacity of ice at -20oC which is 2 kJ/kg/K, (Tsokos KA, 2010), m is the mass of ice and âˆ†T is the temperature difference

(12)

According to Wingsmagazine (2010) Ian Sharkey, the director of de-icing services, with Radiant Aviation Services stated that during an ice storm on March 15-16, 2007 his team had an " average aircraft " block" time (aircraft brakes on to brakes off) of less than 43 minutes for large aircraft" hence it was assumed that the Boeing 747 could be de-iced in this timeframe as well. This information was used to estimate the power that the IR deice would need to bring the ice on an aircraft from -15oC to 0oC in 43 minutes,

(13)

Latent heat required

Next was calculated which is the power needed to melt the ice on the aircraft at 0oC in 47 minutes.

(14)

Hence the total power needed to melt ice on the aircraft in 43 minutes by using infrared deink technology, QIR is

(15)

To make the value of QIR more realistic some assumptions concerning the amount of energy loss, between the heat source and the wing were added. From this the extra power that the laser will need to give out in order to overcome the inferred energy lost as between the source and its destination (transport efficiency) and the energy lost due to reflection by the ice on the surface of the wing (absorption efficiency) was calculated.

Laser device efficiency, Î·laser= 0. 33, Transport efficiency, Î·transport = 0. 75 Absorption efficiency, Î·absorbtion = 0. 75

The laser device, transport and absorption efficiencies reduce the amount of energy that heats the ice. This is called the efficiency loss, 1-Î·i, and it was calculated as follows

,

(16)

Hence,

(17)

Similarly,

(18)

Hence, the percentage extra energy, , required to cover for these energy losses is

(19)