A critical analysis of realistic mathematics education

This essay is a critical analysis of Realistic Mathematics Education (RME) as a cognitive involvement of teaching and learning algebraic linear equations . The learning objective of this essay is to transfer the knowledge of system of algebraic linear equation in students' mind by using contextual problems for their cognitive development. As Freudenthal (1973) declared mathematics as a human activity, so by employing context from real world we will endeavour to achieve our learning aims

The pedagogy is applied on year 7 students . At this stage the students are introduced with algebra first time, so they face a lot of problems in understanding algebraic language (symbols and alphabets) and manipulation of system of linear equations. To comply my learning objective I will employ the unit comparing quantities from mathematics in context curriculum. Simultaneous linear equation is also a part of National Curriculum of England for mathematics at key stage 3 (KS3).

My targeted students are belonging to the working class social background. Majority of them are from different nations. There are 15 students in the class of mixed gender and majority of them are boys. But as Forgasz(2006) argues that girls are better academically over boys in mathematics. The same case is here, in all formative and summative assessments the achieved results of girls are always better. Since the number of students are few in my class so I can easily focus on every student to achieve my targets. There are five students from India, three from china and …….

Algebra :

Algebra is taken as a study of a language and its pattern is a study of procedures for not only solving certain classes of problems but also as an agent for representing generalizations. It is also viewed as the study of principles administrating numerical relations; an idea that focuses on generalization and that can be broadened by including the factors of evidence and demonstration. Algebra has become a focal point of both reform and research efforts in mathematics education (Eric J. Knuth & Ana C. Stephens et al., 2006)

- Is algebra really hard?

Teaching algebra seems to have been a coaching problem since Ancient Times(Radford, 2000). Traditional way of teaching and learning algebra has been a big factor in demise of its popularity. Lack of involvement of students during the class is a common picture. It has been observed that it has reached such heights that students dislike algebra. In worst scenario they even are scared of it.

“ The beginning of Algebra I found far more difficult, perhaps as a result of bad teaching. I was made to learn by heart: The square of the sum of two numbers is equal to the sum of the squares increased by twice their product. I had not the vaguest idea what this meant, and when I could not remember the words, my tutor threw the book at my head, which did not stimulate my intellect in any way”.(The Autobiography of Bertrand Russell 1, p. 36)

So it is proved that the basic problem is not with algebra , it is the problem of traditional teaching methods. At present the situation is not different, still the students are passing out their GCSE without proper understing of algebraic tools. In the result they have totally wrong perception of algebraic tools in the rest of their lives and they are become the part of formal traditional system. Algebra, which is a representative mathematical language, must be graspable for students as it is of vital importance. Being capable of solving algebra can lead the students to manage learning advanced. Thinking logically and analyzing accurately is a skill that a good mathematician can develop easily. Mathematics teaching should promote pupils' ability to solve problems in many spheres . The algebra comprises of following main divisions that are variables, relations, function, equations and inequations (inequalities), and graphs

Mason (1996) states that " The word algebra is derived from the problems of al-jabr (literally, adding or multiplying both sides of an equation by the same thing in order to eliminate negative/fractional terms), which were paralleled by problems of al-muqabala (subtracting the same thing from or dividing the same thing into both sides)." (Wessels, 2009, p. 16)

The equations are the crucial part of algebra and equations are the game of balance. How to create balance between both sides? Here the equal sign '=' plays the important role and normally " many of the difficulties that students have when working with symbolic expression and equations may be attributed to their misconceptions about the meaning of the equal sign." (Stephens et al., 2006, p. 299). Mexicia(2008) is also states that a common mistake among student is misunderstanding the equal sign as a signal for doing something, instead of taking that sign as an operator. generally for students, the equal sign flashes a button in their minds to start working on something rather than its own entity as a “ symbol of equivalence” or “ quantity sameness” . Alibali et al. (1999) argue that “ many elementary and middle school students demonstrate inadequate understanding of the meaning of the equal sign, frequently viewing the symbols as an announcement of the result of an arithmetic operation rather than as a symbol of mathematical equivalence.” (Stephens et al. , 2006, p. 298). He further explains by give an example from six grade students’ i. e. 8 + 4 = â-¡ +5 (falkner et al. 1999)and says that it was found by Falkner et al. that “ many students provided answers of 12, 17, or 12 and 17 – answers that are consistent with an understanding of equal sign as announcing a result” Alibali (1999)also states the similar statement that “ students added all the numbers in the equation or added all the number before the equal sign, again indicating an operational view of the equal sign” (Stephens et al., 2006, p. 298).

The second issues is the lack understand of all algebraic operations before manipulating the system of linear equations. Majority of the students at this level have problems to manipulate the linear equations because this is the first time they are introducing with algebra so they have lack knowledge of algebraic terms specially the letters , Moreover they have weak understanding of equal sign. To overcome this problem the “ Contemporary mathematics curricula try to offer students some help to develop the algebraic ideas and to acquire and make sense of signs. For example, in the new Ontario Curriculum of Mathematics (Ministry of Education and Training 1997), students are introduced to a kind of ‘ transitional’ language prior to the standard alphanumeric- based algebraic language and are asked to find the value of ‘ * ‘ in equations like: * + * + 2 = 8 or the value of ‘ â-¡ ‘ in equations like 32 + â-¡ + â-¡ = 54. This ‘ transitional language’ approach, as any pedagogical approach for teaching algebra, relies on specific conceptions about what signs represent and the way in which the meaning of signs is elaborated by the students. (Radford, 2000, p 239)

Another major problem for the students is how to manipulate equation. Wassels(2009) states some points which are necessary for the learner to understand before manipulating the systems of linear equation. These things are as fellows

“ 1. number sense and operations;

2. properties of operations;

3. concept of a variable;

4. algebraic terms and expressions-(manipulation of algebraic expressions);

5. identifying and expressing relationships.

6. proper interpretation of concept of equation/equality;

7. ability to read and interpret the symbolic form of an equation; and

8. ability to identify appropriate strategies for solving the equations.

9. ability to formulate equations from context problems” (p. 23)

As our topic is system of linear equations having two variables and our aim is to develop the proper concept of equation in students’ mind and it will be achieved by formulating the equations from context problems and then solving those equations and eventually find out the solutions of the realistic life problems so we will discuss about some informal and pre-formal methods for solving the contextual problem. Normally in primary classes the students have already the knowledge of word problems but they are not familiar with the algebraic terms, so if students are encouraged to apply any known method then they will take more interest and this is the best way to enhance their cognitive development.

“ Graeber & Tanenhaus (1993) says that “ It has been found from the numerous studies that when students are working on making the algebraic equation they do not realize the situation modelled in the problem when determining which operation to apply , so they select the operation by guessing, by trying all operation and choosing one that gives what seems to them a reasonable answer, or by studing such properties as the size of the numbers involved.” (schifter, 1999, p. 63)

Realistic Mathematics Education:

Freudenhal (1973) realised the misunderstanding of the students with mathematics and he started work against the traditional teaching methods and try to familiarize the new generation the proper concept of mathematics. Freudenthal (1973) announced Mathematics as a human activity.

Realistic Mathematics Education

The development of the RME (Realistic mathematics education) evolved after thirty years of developmental research in teaching and learning mathematics in the Netherlands and is primarily based on Freudenthal’s interpretation of mathematics as a human activity (Freudenthal, 1973; and Gravemeijer, 1994).

Philosophy of Realistic mathematics Education

Realistic Mathematics Education (RME) was the Dutch effort to change and reform the teaching of Mathematics all around the world in 1960s. It was developed by using extensive research and mainly uses the ideas of Freudenthal , Gravemeijer and Treffers. (carol Marshall 2003). This theory was developed by Wiskos project in Netherlands. The present form of RME is mostly determined by Freudenthals views on mathematics. Two of its major points are that mathematics must be connected to reality and it should be seen as a human activity (zulkardi, 2002). Freudenthal ideas on the process of Mathematisation were that pupils should be involved in guided reinvention of Mathematics which in other words mean that they develop Mathematics themselves as mathematicians did before them , beginning with informal strategies and then gradually moving to more formal strategies. (Carol Marshall, 2003). Thus in RME context is used to help students to understand Mathematics which is in keeping with Freudenthals theory that mathematics must be connected to reality and in order to be of human value it should stay close to children and be relevant to society. (Carol Marshall, 2003). Later on, Treffers (1978, 1987) gave the idea of two types of mathematization in an educational context and differentiated “ horizontal” and “ vertical” mathematization which will be described later.

Misunderstanding of ‘ realistic’

In RME, the real world is used as a starting point for the development of mathematical concept and ideas. Real world is the rest of the world outside mathematics, i. e., school or university subjects or disciplines different from mathematics, or everyday life and the world around us (Hadi , 2002). One thing which is important to understand is that the word realistic does not necessarily means that it should be connected to the real world but also to the problem situations which are real in pupils mind.

The realistic approach Vs the mechanistic approach

In the conventional (mechanistic) style of the learning process the teachers take control over each activity. In contrast, the RME approach suggests that the pupils are supposed to take responsibility for their own learning and actively engage in interactive discussion in the classroom. Guided by the teacher, the pupils reinvent informal and formal mathematics models in a process of mathematizing contextual problems(Armanto, 2002).

A notable difference between RME and the traditional approach to mathematics education is the rejection of the mechanistic, procedure-focused way of teaching in which the learning content is split up in meaningless small parts and where the students are offered fixed solving procedures to be trained by exercises, often to be done individually. RME, on the contrary, has a more complex and meaningful conceptualization of learning. The students, instead of being the receivers of ready-made mathematics, are considered as active participants in the teaching-learning process, in which they develop mathematical tools and insights (Heuvel-Panhuizen, 1998). In this view mathematics education would be highly interactive in which the teachers would have to build upon the ideas of the students. It means they have to react based on what the students bring to the fore (Kooj, 1999).

RME’S KEY PRINCIPLES

According to Gravemeijer (1994, 1997) for instructional design there are three key principles of RME namely

guided reinvention through progressive mathematization,

didactical phenomenology

self developed models or emergent models.

Guided reinvention through progressive mathematization

According to de Lange (1987), in RME the real world problem is explored in the

first place with the view to mathematizing it. This means organizing and

structuring the problem, trying to identify the mathematical aspects of the problem

to discover regularities (Fauzan).

In the guided reinvention principle, the students should be given the opportunity to

experience a process similar to that by which mathematics was invented

(Gravemeijer 1994, 1999). According to this principle a learning route has to be

mapped out that help the pupils to find the intended mathematics by themselves.

‘ Mathematizing’ is a very important activity in RME. This activity mainly involves generalizing and formalizing (Gravemeijer, 1994). Formalizing includes modeling, symbolizing, schematizing and defining, and generalizing is to understand in a reflective sense. By solving the contextual problems in realistic approach students learn to mathematize contextual problems.

This process is called mathematization (Treffers, 1987, 1991a).

In words of Freudenthal (1971), the activity that we perform in RME is:

An activity of solving problems, of looking for problems, and also an activity of organizing a subject matter. This can be a matter from reality, which has to be organized according to mathematical patterns if they have to be solved. It can also be a mathematical matter, new or old results, of your own or others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach.

This organizing activity is called ‘ mathematizing’ (Gravemeijer, 1994)

Mathematization is a process from the real model into mathematics . Modeling or model building is the entire process from the original real situation to a mathematical model (Blum & Niss, 1989).

De Lange (1996) described the process of conceptual mathematization in

RME. According to him the process of developing mathematical concepts and ideas starts from the real world, and at the end we need to reflect the solution back to the real world. In other words in mathematics education things are taken from real world , are methamitised and then brought back to real world. (Fauzan, 2002)

All this process lead to conceptual mathematization.

RME can be distinguished from other theories in mathematics education such as

mechanistic, empiristic, and structuralist according to the presence or absence of

the components of horizontal and vertical mathematization (Treffers, 1991).

Treffers (1987, 1991) distinguishes two types of mathematization, i. e. vertical and

horizontal, which are described by Gravemeijer (1994) as reinvention process.

In a horizontal mathematization the students come up with mathematical tools which can help to solve a problem located in real life situation (Merja van den Heuvel-Panhuizen , 1998)or in other words tries to describe the problems using own language or symbols, and solves the problems. In this process it is possible that each learner has his or her own way which is different form others' solution.

In vertical mathematization it is also started by contextual problems, later on the learners can construct certain procedure that can be applied to the similar problems directly, not necessarily using a bridge of the context.

Thus in words of Gravemeijer (1990, 1994) vertical mathematization is actually a mathematization of mathematical matter, whereas horizontal mathematization is indeed the mathematization of contextual problems.

Thus briefly in words of Freudenthal (1991) – " horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols."(Van den Heuvel-Panhuizen, M. 1998)

Freudenthal (1991) refers to processes of vertical and horizontal mathematization where horizontal is developing tools to solve a problem presented in a real life context and vertical is making connections between concepts and strategies, moving within a more formal world of symbols. The shift in mathematical thinking from ' model of' to ' model for' became a significant element of RME thinking in the early 1990s when this crucial shift was detected (Carol Marshall 2003).

b) Didactical Phenomenology

In contrast to the anti-didactic inversion, Freudenthal (1983) advocated the didactical phenomenology. This means that in learning mathematics one should start from phenomena that are meaningful for the student and thus stimulate learning processes. In didactical phenomenology, situations where a given mathematical topic is applied are to be investigated for two reasons (Gravemeijer, 1994, 1999). Firstly, to reveal the kind of applications that have to be anticipated in instruction. Secondly, to consider their suitability as points of impact for a process of progressive mathematization.

According to Gravemeijer (1994, 1999), the goal of a phenomenological

investigation is to find problem situations for which situation-specific approaches

can be generalized, and to find situations that can evoke paradigmatic solution

procedures that can be taken as the basis for vertical mathematization. This goal is

derived from the fact that mathematics is historically evolved from solving practical

problems. In mathematics instruction we can realize this goal by finding the

contextual problems that lead to this evolving process.

In the concept of didactical phenomenology Freudenthal refers to the world from which we have abstracted. Thus this concept focuses on the connections between a mathematical idea/concept and the world which relates to it . These connections form the phenomenology of the mathematical structure. The implications of that world for the instruction of students is the didactical phenomenology.

c) Self-developed models

The third key principle for instructional design in RME is self developed models (Gravemeijer 1994, 1999). This principle plays an important role in bridging the gap between informal knowledge and formal knowledge. According to it we have to give the opportunity to the students to use and develop their own models when they are solving the problems. To begin with the students will develop a model which is familiar to them. After the process of generalizing and formalizing, the model gradually becomes an entity on its own. Gravemeijer (1994) calls this process a transition from model-of to model-for. After the transition, the model may be used as a model for mathematical reasoning (Gravemeijer, 1994,

1999; Treffers, 1991a).

Characteristics or Tenets of RME

The combinations of three Van Hiele's levels, Freudenthal's didactical phenomenology and Treffer's progressive mathematization result in five basic characteristics of realistic mathematics education or five tenets of RME (de Lange, 1987, Gravemeijer, 1994).

The use of contexts.

The use of models.

The use of students’ own productions and constructions.

The interactive character of the teaching process.

The intertwinement of various learning strands.

(1) Use of contextual problems (contextual problems figure as application and as

starting points from which the intended mathematics can come out).

(2) Use of models or bridging by vertical instruments (broad attention is paid to

development models, schemas and symbolization rather than being offered the rule or formal mathematics right away).

(3) Use of students’ contribution (large contributions to the course are coming from student’s

own constructions, which lead them from their own informal to the more standard formal methods).

(4) Interactivity (explicit negotiation, intervention, discussion, cooperation and evaluation among pupils and teachers are essential elements in a constructive learning process in which the student’s informal strategies are used as a lever to attain the formal ones).

(5) Intertwining of learning strands (the holistic approach implies that learning strands can not be dealt with as separate entities; instead an

intertwining of learning strands is exploited in problem solving).(Zulkardi 1999)

Principles and Characteristics of RME

The characteristics of RME are historically related to three Van Hiele's levels for of learning

mathematics(de Lange, 1996). Here it is assumed that the process of learning proceeds

through three levels: (1) a pupil reaches the first level of thinking as soon as he can

manipulate the known characteristics of a pattern that is familiar to him/her; (2) as soon as

he/she learns to manipulate the interrelatedness of the characteristics he/she will have reached

the second level; (3) he/she will reach the third level of thinking when he/she starts

manipulating the intrinsic characteristics of relations.

Traditional instruction is inclined to

start at the second or third level, while realistic approach starts from first level. Then, in order to start at the first level that deals with phenomena that are familiar to the students,

Freudenthal’s didactical phenomenology that learning should start from a contextual

problem, is used. Furthermore, by the guided reinvention principle and progressive

mathematizations, students are guided didactically and efficiently from one level to another

level of thinking through mathematization. These two principles and the concept of self

developed models (Gravemeijer, 1994) can be used as design principles in the domainspecific

instruction theory of mathematics education. The LE used these learning principles

both in the course and the web site.

Developing tools for manipulating systems of linear equations by using RME in MiC:

Our aim is to change the traditional concept of algebra and use context from real life to make the proper concept of mathematics in students’ mind.

The Dutch experiences in research and development in mathematics education of the past decades, the philosophy of RME, and the NCTM Standards form also the base for the approach towards algebra in the MiC curriculum. In this curriculum, algebra is characterized as:

"… the study of relationships between variables (the study of joint variation). Students learn how to describe these relationships with a variety of representations, and will be able to connect the representations. The algebra is used to solve problems, and students learn how to use algebra in an appropriate manner. The latter includes making intelligent choices about what algebraic representation (if any) to use in solving a problem. Algebra (its structure and symbols) is not a goal on itself. Algebra is a tool to solve problems. The problems that arise from the real world, are often presented in contexts, they are realistic problems in the way described earlier."

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