Free critical thinking on math help

1. Find the equation of the tangent to:

a) u = x – 2x2 + 3 at x = 2

u1= 1 – 4x

Gradient = 1 – 4(2)

= - 7

The equation of a line is of the form y = mx + c , m is the gradient and c is the y intercept.

When x = 2,

y = 1 – 2(2 x2) + 3

y = 1 – 8 + 3

y = -4

Thus the equation of the tangent line will be given by,

Y = -7x + c substituting the point (2, -4) in the equation we have;

-4 = -7 x 2 + c

-4 + 14 = c

10 = c

y = -7x + 10

b. y = x1/2 + 1 at x = 4

y1 = 1/2 x-1/2

Gradient = 1/2 (4)-1/2

= 1/2 x 1/2

= 1/4

Equation will be given by; y = 1/4x + c

When x = 4, y = 41/2 + 1

Y = 3

The value of c is given by;

3 = 1/4(4) + c

C = 3 – 1

C= 2

Therefore, the equation of the tangent line will be;

Y = 1/4 x + 2

c). y = 5/x1/2 – x1/2 at point (1, 4)

y1= -5/2x3/2 – 1/2 x-1/2

Gradient = -5/2 x 13/2 – 1/2 (1)-1/2

Gradient = -2

The equation of the tangent at point (1, 4) will be given by;

y = -3 x + c, substituting the point (1, 4) to find c

4 = -3(1) + c

C = 8

The equation of the tangent will be;

y = -3x + 8

d). y = 8x1/2 – 1/x2 at x = 1

y1 = 4x-1/2 + 2/x3

Gradient = 4 x (1)-1/2 + 2 /13

Gradient = 6

When x = 1, y = 8(1)1/2 – 1/12 = 7

We are finding the equation of the tangent line at the point (1, 7)

y = 6x + c

7 = 6(1) + c

C = 1

The equation of the tangent line will be;

y = 6x + 1

5. Find the equation of the tangent to:

a) y = (2x + 1)1/2 at x = 4

y1 = 1/2 (2x + 1)-1/2(2)

y1 = (2x + 1)-1/2

Gradient = (2 x 4 + 1)-1/2

Gradient = 9-1/2

= 1/3

At x = 4, y = (2 x 4 + 1)1/2 = 3

We are finding the equation of the tangent line at the point (4, 3)

y = 1/3 x + c, substituting the point (4, 3) in the equation we have;

3 = 1/3 x 4 + c

C = 3 – 4/3

C = 5/3

The equation of the tangent will be;

y = 1/3 x + 5/3

c. f(x) = x/1-3x at (-1, -1/4)

f1(x)=[x1(1- 3x) – x (1 -3x)1]/ (1 – 3x)2

f1(x)= 1 – 3x – x (-3)/ (1 – 3x)2

f1(x)= 1/(1 – 3x)2

Gradient = 1/ (1 – 3 x -1)2

Gradient = 1/16

The equation of the tangent line at (-1, -1/4) will be given by;

Y = 1/16 x + c

-1/4 = 1/16 (-1) + c

C = - 3/16

The equation will be; y = 1/16x – 3/16

6. Find the equation of normal to:

a. y = 1 / (x2 + 1)2

y1 = - 2(x2 + 1) x (2x)/ (x2 + 1)4

y1 = -4x (x2 + 1)/ (x2 + 1)4

y1 = -4x / (x2 + 1)3

Gradient = - 4(1)/ (1 +1)3

Gradient = - 4 / 8

= -1/2

The equation of the normal at point (1, 1/4) will thus be given by;

y = -1/2 x + c

1/4 = -1/2(1) + c

C = 1/4 + 1/2

C = 3/4

Y = -1/2 x + ¾

c. f(x) = x1/2(1- x)2 at x = 4

f(x)1 = 1/2 x-1/2(1- x)2 + 2x1/2(1- x)(-1)

f(x)1 = 1/2 x-1/2(1- x)2 - 2x1/2(1- x)

Gradient = 1/2(4)-1/2(1 – 4)2 – 2(4)1/2(1 – 4)

= 57/4

y = 57/4x + c

At x = 4 , y = 41/2 (1 – 4)2 = 18

The equation of the normal at point (4, 18) will be given by;

y = 14x + c

18 = 57/4 x 4 + c

C = 18 – 57

C = -39

Therefore the equation of the normal will be 4y = 57x - 156

3a. Find the equations of the horizontal tangents to y = 2x3 + 3x2 – 12x + 1

The gradient of the horizontal tangent to a curve is always 0

y1 = 6x2 + 6x – 12

0 = 6x2 + 6x – 12

Dividing through by 6 we have

0 = x2 + x – 2

Product -2 and sum is 1. The numbers are 2 and -1

x = -2 or x = 1

When x = -2, y = 2 (-8) + 3(4) – 12(-2) + 1

y = 21

the point is (-2, 21)

Since the gradient is zero, then the equation will be y = 21

When x = 1, y = 2(1) + 3(1) – 12(1) + 1

y = -6

Thus the point is (1, -6)

Since the gradient is zero, the equation will be y = -6

b. Find the points of contact where horizontal tangents meet the curve y = 2x1/2 + 1/ x1/2

for horizontal tangents of a curve, the gradient is zero. Therefore

y1 = -x-1/2 – ½ x-3/2

0 = -1/x1/ 2 – 1/2x3/2

1/x1/ 2 = -1/2x3/2

1 = -1/2x

2x = -1

X =-1 /2

At x = -1/2 , y = 2(-1/2)1/2 + 1 / (-1/2)1/2

c. Find k if the tangent to y = 2x3 + kx2 – 3 at the point where x = 2 has gradient4

y1 = 6x2 + 2kx

4 = 6(2x2) + 2(2)k

4 = 24 + 4k

-20 = 4k

K = -5

d Find the equation of the tangent to y = 1 – 3x + 12x2 – 8x3 which is parallel to the tangent at (1, 2)

y1 = -3 + 24x – 24x2

Gradient at the point (1, 2) = -3 + 24 x 1 – 24 x 1

= -3

This is the gradient of the tangent line parallel to the tangent of the line in question.

Since the gradient of two parallel lines is the same, then the gradient of the line in question = -3

Therefore,

-3 = -3 + 24x – 24x2

0 = 24x – 24x2

0 = 24x (1 – x)

1 – x = 0 or 24x = 0

X = 1 or x = 0

When x = 1, y = 1 – 3 + 12 – 8 = 2

When x = 0, y = 1

The equation of the tangent line at the point (1, 2) will be;

y = -3x + c

2 = -3 + c

C = 5

y = -3x + 5

On the other hand, the equation of the tangent line at the point (0, 1) will be;

y = -3x + c

1 = 0 + c

1 = c

y = -3x + 1

7. y = a (1 – bx)1/2 where a and b are constants, has a tangent with the equation 3x + y = 5 at the point where x = -1. Find a and b.

y1=-1/2ab (1 – bx)1/2

-3 = -1/2ab(1 + b)-1/2

6 = ab(1 + b)-1/2(i)

When x = -1,

y = a(1 + b)1/2(ii)

y = - 3x + 5, but x = -1 so, y = 3 + 5

y = 8

Therefore, equation (ii) becomes

8 = a(1 + b)1/2..(iii)

Substituting equation (iii) in to equation (i) we have,

-6 = ab x a / 8

-48 = a2b

8 a. Find where the tangent to the curve y = x3 at the point where x = 2, meets the curve again.

y1 = 3x2

Gradient at x = 2

Gradient = 3(2 x 2)

= 12

When x = 2, y = 2 x 2 x 2 = 8

The tangent at point (2, 8) of the curve will be;

y = 12x + c

8 = 24 + c

-16 = c

y = 12x – 16

The tangent will be the same at all points where it meets with curve since it is a straight line.

b. Find where the tangent to the curve y = -x3 + 2x2 + 1 at the point where

x = -1, meets the curve again.

y1 = -3x2 + 4x

Gradient at x = -1

Gradient = -3(1) – 4

= -7

At x = -1, y = -(-1) + 2(-1) + 1

y = 0

The tangent equation will be

y = -7x – 7

The tangent will be the same at all points where it meets with curve since it is a straight line.

c. Find where the tangent to the curve y = x3 + 4 / x at the point where x =

1, meets the curve again.

y1= 3x2 – 4 / x2

Gradient = 3(1) – 4

= -1

At x = 1, y = 1 + 4

y = 5

The tangent at the point (-1, 5) will be given by;

y = -x + c

5 = 1 + c

C= 4

y= -x + 4

The tangent will be the same at all points where it meets with curve since it is a straight line.