Laplace transformations and their application

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INTRODUCTION

Pierre-Simon Laplace (1749-1827)

Laplace was a French mathematician, astronomer, and physicist who applied the Newtonian theory of gravitation to the solar system (an important problem of his day). He played a leading role in the development of the metric system.

The Laplace Transform is widely used in engineering applications (mechanical and electronic), especially where the driving force is discontinuous. It is also used in process control.

This subject originated from the operational method applied by the Engineer Oliver Heaviside (1850-1925), to problems in electric engineering. Unfortunately, Heaviside’s treatment was unsystematic and lacked rigour, which was placed on sound mathematical footing by Bromwich and Carson during 1916-17. It was found that Heaviside’s operation calculus is best introduced by means of particular type of definite integrals called Laplace Transforms.

It is always useful, and often essential, to analyse the performance capabilities and the stability of a proposed system before it is build or implemented. Many analysis techniques centre around the use of transformed variables to facilitate mathematical treatment of the problem. In the analysis of continuous time dynamical systems, the use of Laplace Transforms predominates.

Applying Laplace Transforms is analogous to using logarithms to simplify certain types of mathematical manipulations and solutions. By taking logarithms, numbers are transformed into powers of 10 or some other base, e. g. natural logarithms. As a result of the transformations, mathematical multiplications and divisions are replaced by additions and subtractions respectively. Similarly, the application of Laplace Transforms to the analysis of systems which can be described by linear, ordinary time differential equations overcomes some of the complexities encountered in the time-domain solution of such equations.

DEFINATION:

Laplace Transforms are used to convert time domain relationships to a set of equations expressed in terms of the Laplace operator ‘ s’. Thereafter, the solution of the original problem is effected by simple algebraic manipulations in the ‘ s’ or Laplace domain rather than the time domain. The Laplace Transform of a time variable f (t) is defined as:

F(s)= L {f(t)} = &integral; f (t) e dt

where *L{.}* is used to denoted the transformation.

BASIC PROPERTIES OF THE LAPLACE TRANSFORM

The following are some of the fundamental properties of Laplace Transforms:

P1) The Laplace Transformation is linear, i. e.

L {f1(t) + f2(t)}= L{f1(t)} + L{f2(t)} = F1(s) + F2(s)

and L{kf (t)} = kL{f (t)}= kF(s) k = constant

P2) Laplace Transformations of derivatives are given by the following:

L{df (t) / dt}= L{f′(t)} = sF(s) – f (0)

where f (0) is the initial value of f (t) , at t = 0.

L{d f (t)/dt}= L{f″t)}= s F(s) – sf (0) – f′(0)

In general,

L{d f (t)/ dt}= L{f (t)}= s F(s) – s f (0) -…- f (0)

P3) Laplace Transforms of integrals are given by:

L{f (t)}= [F(s) – f (0)]/s

In general,

L{f – n (t)}= F(s) / sn + f – 1 (0) / sn + f -2 (0) / sn-1 +!+ f -n (0) / s

P4) The ‘ Final Value’ theorem states that:

lim f(t) = limsF(s)

t→∞ s→0

and facilitates the determination of the value of f (t) as time tend towards infinity, i. e. the steady-state value of f (t).

P5) The ‘ Initial Value’ theorem states that:

lim f(t) = limsF(s)

t→0 s→∞

and allows the determination of the value of f (t) at time t = 0+ , i. e. at a time instant immediately after time t = 0.

Properties P1 to P4 are the most often used in systems analysis.

To return to the time-domain from the Laplace domain, *inverse Laplace Transforms* are used. Again this is analogous to the application of anti-logarithms and as in the use of logarithms, tables of Laplace Transform pairs help to simplify the task.

INVERSE LAPLACE TRANSFORM

Definition:

If, for a given function F(s), we can find a function f(t) such that L(f(t)) = F(s), then f(t) is called the inverse Laplace transform of F(s).

Notation: f(t) = L (F (s)).

Hence to find the inverse transforms, we first express the given function of s into partial fraction which will, then, be recognizable as standard forms.

TRANSFORM OF SHIFTS IN ‘ s’ AND ‘ t’

1. If L(f(t)) = F(s), then L(e f(t)) = F(s – a) for any real constant a.

Note that F(s – a) represents a shift of the function F(s) by a units to the right.

0. The unit step function s(t) = 0 , ha t < 0 es s(t) = 1 , ha t – 0:

If a > 0 and L(f(t)) = F(s), then L(f(t – a) · s(t – a)) = F(s)e.

TRANSFORM OF POWER MULTIPLIERS

If L(f(t)) = F(s), then

L(t f(t)) = (-1) d F(s)

ds

for any positive integer n, particularly L(tf(t)) = (-1)F′(s)

CONVOLUTION

Definition:

Given two functions f and g, we define, for any t > 0,

(f * g)(t) = ∫ f(x) g(t – x) dx

The function f * g is called the convolution of f and g.

Remark The convolution is commutative.

Theorem (The convolution theorem)

L((f * g)(t)) = L(f(t)) · L(g(t)).

In other words, if L(f(t)) = F(s) and L(g(t)) = G(s),

then L (F(s)G(s)) = (f *g)(t).

LAPLACE TRANSFORM OF A PERIODIC FUNCTION

Definition:

A function f is said to be periodic if there is a constant T > 0 such that f(t + T) = f(t) for every t. The constant T is called the period of f.

The sine and cosine functions are important examples of periodic function. One other example is the periodic triangular wave. It is is the function defined by f(t) = t if

0 <= t <= 1 , f(t) = 2 – t if 1 <= t <= 2 and f(t + 2) = f(t) for any t.

The following proposition is useful in calculating the Laplace Transform of a periodic function.

Proposition:

Let f be a periodic function with period T and f1 is one period of the function, Then (as usual F(s) = L(f(t))):

F(s)= L(f1(t)) = 1 ∫e f(t) dt.

THE UNIT STEP FUNCTION

At times, we come across such fractions of which the inverse transform cannot be determined from the formulae so far derived. In order to cover such cases, we introduced the unit step function(or Heaviside's unit function)

Definition:

The unit step function, u(t), is defined as

That is, *u* is a function of time *t* , and *u* has value zero when time is negative (before we flip the switch); and value one when time is positive (from when we flip the switch).

Shifted Unit Step Function

In many circuits, waveforms are applied at specified intervals other than t = 0. Such a function may be described using the shifted (aka delayed) unit step function.

Definition of Shifted Unit Step Function:

A function which has value 0 up to the time $t = a$ and thereafter has value 1, is written:

APPLICATIONS

The Laplace transform is used frequently in engineering and physics; the output of a linear time invariant system can be calculated by convolving its unit impulse response with the input signal. Performing this calculation in Laplace space turns the convolution into a multiplication; the latter being easier to solve because of its algebraic form.

The Laplace transform can also be used to solve differential equations and is used extensively in electrical engineering. The method of using the Laplace Transform to solve differential equations was developed by the English electrical engineer Oliver Heaviside.

REFERENCES

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