# Bond pricing and interest rates 

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## Bond Pricing And The Term Structure Of Interest Rates: A New Methodology For Contingent Claims Valuation

We read the paper Bond pricing and the term structure of interest rates by Heath, Jarrow, and Morton. Their paper presents a theory for valuing contingent claims under a stochastic term structure of interest rates. The methodology takes as given an initial forward rate curve and a family of potential stochastic processes for its subsequent movements. A no arbitrage condition restricts this family of processes yielding valuation formulae for interest rate sensitive contingent claims which do not explicitly depend on the market prices of risk.

In relation to the term structure of interest rates, arbitrage pricing theory has two purposes. The first is to price all zero coupon (default free) bonds of varying maturities from a finite number of economic fundamentals, called state variables. The second, is to price all interest rate sensitive contingent claims, taking as given the prices of the zero coupon bonds.

The primary contribution of this paper, however, is a new methodology for solving the second problem, i. e., the pricing of interest rate sensitive contingent claims given the prices of all zero coupon bonds.

The methodology is new because (i) it imposes its stochastic structure directly on the evolution of the forward rate curve, (ii) it does not require an " inversion of the term structure" to eliminate the market prices of risk from contingent claim values, and (iii) it has a stochastic spot rate process with multiple stochastic factors influencing the term structure. The model can be used to consistently price (and hedge) all contingent claims (American or

European) on the term structure, and it is derived from necessary and (more importantly) sufficient conditions for the absence of arbitrage.

## The Difference Between Duration And Maturity In Bonds

- Based on the article: Bond Price Volatility and Term to Maturity: A generalized Re-specification

Most investors and especially we who are taking the course Fixed Income Securities are familiar with the bonds maturity. The article wants to illustrate why we can't just rely on the length of maturity when estimating how volatile the price for a certain bond is. As we have read before during the course, there is a common and accepted thumb rule that tells us that for a given change in yields, the price change for the bond will be greater the longer the term to maturity is. Therefore I first of all want to highlight the difference between the duration and maturity. Firstly I will have a short explanation of these two terms and further I will continue this paper by explain some important parts from the article.

When it comes to maturity, we all know the maturity is the point in time when the investor receives back the principal. We also know that a bond will increase in value, that is, the price of the bond will increase, if the interest on the market decrease and vice versa. From this statement above, it may be clear that the longer maturity, the more changes in the interest rates can be waited and the more volatile the bond price will be.

The duration of a bond will show how sensitive a bond's price is to changes in the interest rate. It's a measurement for how much the bond price will change due to a one percentage change in the interest rate on the market.

Duration of 7 means for instance that if the interest rate raises by one percentage means that the price of the bond will fall 7 percentages. The duration is simply the weighted average amount of time that it takes for the investor to be repaid all cash that is both the coupon payments and the end, principal payment. Therefore the duration will always be less than the maturity, except for zero-coupon bonds where they will be equal. These two properties are important when it comes to duration:

- The first one is that the longer the maturity, the higher the duration.
- The second one is that the lower the coupon payment the higher the duration.

With these facts above, I want to highlight the important aspect of this what the article goes through. Indeed, the price volatility is connected to the time structure of the bond, but it's not direct mathematically related to the maturity in a pure simple way. Since there are evidence that duration is more accurate, the authors for the article wants to generalize the following:

- For a given basis point change in market yield, percentage changes in bond prices vary proportionally with the duration and are greater, the greater the duration of the bond.
- There is also true that there is an inverse relationship between duration and coupon. This means that a higher coupon bond will automatically be seen as a shorter-term bond than a bond that has a lower coupon payment, even if they in fact have the same maturity period. Furthermore, this means that a comparison of these bonds with equal maturity will underestimate the default risk premium in periods of upward sloping yield curves and also overestimate the premium in
periods of downward sloping yield curves. For instance, referred to the above information, there is evidence that the duration varies inversely with coupon rates. A 50 year 8 percent coupon bond, yielding 6 percent, have approximately the same duration as a 20 year 2 percent coupon bond yielding the same amount of 6 percent.

To sum up the article and this paper, I want to highlight the complexity of the relationship between the bond price volatility and the maturity, as well as the relation between then bond value and the duration, even if I believe that duration is a more accurate measurement of price volatility.

## Expectations, Bond Prices, And The Term Structure Of Interest Rates

 The term structure of interest rates is of great importance when dealing with bonds, since the interest rate significantly affects the bond price. Burton G. Malkiel examines the relationship between market interest rates and bond prices in his article Expectations, Bond Prices, and the Term Structure of Interest Rates, where he takes the position that Lutz theory of Basic Behavioural postulate is correct and important in understanding the behaviour of market interest rates of securities with different term to maturity. Lutz says that investors decide whether to invest in bond based on their expectation of future short rates, since they are not able to predict long term rates. Malkiel furthermore aims to ease the principle hypothesis by Hicks and Keynes, that future prises are biased expectations of future spot prices.A bond's market price or value is determined by four factors: the face value of the bond; the coupon or interest paid periodically to the bondholder; the effective interest rate per period; and the number of years to maturity. The
lower interest rate, the higher the bond price - hence the significant relationship between the two factors. The term structure in this sense is important since the investor wants to choose the term and bond that is most beneficial. According to Malkiel, the term structure is based on investors' expectations, which is influenced by the normal range of interest rates. If interest rates appear to be very high relative to the normal range, investors may expect that interests will fall and vice versa.

The term structure problem is furthermore analysed with a combination of spot and forward trading, resulting in longer term rates as combinations of relevant forward short rates:
$(1+R 2) 2=(1+r 1)(1+r 2)$. When long-term average rates are below the current short rate future short-term rates are expected to fall, and conversely, long rates will exceed the current short rate if future short rates are expected to rise. Additionally, when interest rates are believed to be high in relation to historical averages, investors will prefer long-term bonds while issuers prefer to sell short-term securities, whilst low interest rates will encourage investors to buy shorts and issuers to sell longs.

Supporting Lutz's theory, Malkiel makes the conclusion that short and intermediate areas of the yield curve exhibit more dramatic responses to changes in expectations. This is due to the fact that investors cannot predict the long term rates; they only interpret the near past and current market conditions. What is also worth mentioning is that the term structure fluctuates more between e. g. one to two years, and three to six years, compared to a very long period of time, e. g. thirty-four and sixty-eight
years. This is also due to the fact that it is difficult to predict changes in interest rates for such a far away future. This furthermore explains that the yield curve tends to flatten out the longer term to maturity.

To conclude, investors will choose to purchase bonds depending on their expectations of how interest rates will change in the near and long-term future, the term-structure, and that presumably would be most beneficial in terms of bond price and returns.

## Does Duration Extension Enhance Long-Term Expected Returns?

The article's main purpose is to give investors important information regarding duration and if you can gain a profit or not in the long-term. The author is using empirical evidence mainly from the U. S. Treasury bond market over the past 25 years. All the results of the past returns depend on the interest rate trend in the period the authors are looking at. The focus in the article lies on the long-run expected return differentials across bonds with different maturities. The risk premium is defined as the long-term return exceeding short-term risk-less rate.

The writer means that the one-year bill earns on average 150 basis points higher return than one-month bill and after two years the yield curve will remain a constant line. In other words the return of the bill will stay reasonably the same at two years but recall that long-term bonds are riskier than short-term bonds because it is difficult to predict the future. In other words it is uncertain how much the bonds are worth in the long-term because there are many different causes that affect the value in the future.

The article discusses the bond risk premium using six theories. There are three classic term structure hypotheses. The first is called pure expectations which means assuming that there is no risk premium. The second is the liquidity/risk hypothesis explaining the compensation for return volatility. The third explains the increase and decrease with duration depending on time horizon called the preferred habitat theory.

Ilmanen presents two modern asset pricing theories. One that explains the risk premium proportional to return volatility and the other one that clarifies CAPM. The latter explains that the risk of assets depends on the sensitivity to aggregate wealth as in stock market sensitivity ( $\beta$ correlation) and risk premium ( $\beta$ Market risk premium) which in turn depends on market volatility and risk aversion level.

Equilibrium model means assets performing poorly in bad times should earn positive risk premium while assets performing well are accepted for low yields but other non-risk related factors are also mentioned.

To sum up the article long-term bonds are riskier than short-term bonds and investors earn positive risk premium for bearing this risk. Various models specify that expected returns are linear in duration and return volatility but other factors may contribute.

## References

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