# Bond valuation assignment

Assignment no. 1 Fixed Income Securities and Markets Question A. 1 Given the following bond: | starting date | 30/09/2011 | | maturity date | 30/09/2014 | | coupon rate | 4. 00% | | coupon frequency | annual | | day count | act/act | | nominal value | 100 | a) Calculate the price of the security on the 30/09/2011, if the yield to maturity is 5% (NB: Price= PV of future cash flows). ) Given the price and the yield to maturity of the bond, calculate the three components of the (expected) total return of this investment (if you invest 100 Euro). c) What will be the price of this bond after one year (on 30/09/2012) if its yield to maturity is 3%? d) If on 30/09/2012 you decide to sell the bond at the price calculated in the previous question, what will be the return of your investment? How this differs from the expected return calculated in (b)? Comment. Solution a) Using the pricing formula for bonds: [pic] b) The three different components are: . Coupon: holding the bond until maturity, the investor will receive three coupons of size 4 Euro, therefore the coupon component will be nC= 12 b. Capital gain: it is the difference between the price at maturity and the price at purchase, equal to 100-97. 26= 2. 74 c. The interest on interest is 0. 61, from the following formula: [pic] The total expected return from this investment is therefore equal to 12+2. 74+0. 61= 15. 35. In percentage terms, the return is equal to 15. 35 divided by the invested amount (97. 26), minus 1, or 15. 78%.

In annual term the percentage return will be one third of the period return, i. e. 5. 12%. c) Applying again the formula used in (a): [pic] d) If the bond is sold on 30. 09. 2012 at 3% yield to maturity, the realized return of this investment will be: a. Coupon: the bond has been held for one year, therefore the investor will receive one coupon, equal to 4 b. The capital gain

will be equal to 101. 91-97. 26= 4. 64 c. No interest on interest is gained, because the first coupon is received at the selling date The total realized return of this investment has been 8. 4 Euro, which corresponds to an annual return of 8. 64%. The annual return is higher then the expected return because the second component (capital gain) has been higher than expected. The decrease in the (required) yield to maturity on this bond has caused an appreciation of 4. 64 Euro in one year, while the investor expected a capital gain of 2. 74 in three years. Question A. 2 Suppose that a bond is purchased between coupon periods. The days between the settlement date and the next coupon period is 115. There are 183 days in the coupon period. Suppose that the bond purchased has a coupon rate of 7. % and there are 10 semi-annual coupon payments remaining. a) What is the dirty price for this bond if a 5. 6% discount rate is used? b) What is the accrued interest for this bond? c) What is the clean price? Solution The fundamental pricing formula can be used to find the dirty price of the bond. In particular, the dirty price is equal to the present value of all future cash flows due to the bond holder. The present value of each cash flow is calculated in the table below. Both coupon rate and yield to maturity considered in the calculation are semi-annual: days (from settlemt) | Event | cash flow | yield to maturity | PV | |-68 | last coupon | | | | | 0 | settlement date | | | | | 115 | coupon | 3. 7 | 2. 80% | 3. 63617 | | 297 | coupon | 3. 7 | 2. 80% | 3. 53686 | | 479 | coupon | 3. | 2. 80% | 3. 44027 | | 661 | coupon | 3. 7 | 2. 80% | 3. 34682 | | 843 | coupon | 3. 7 | 2. 80% | 3. 25541 | | 1025 | coupon | 3. 7 | 2. 80% | 3. 16698 | | 1207 | coupon | 3. 7 | 2. 80% | 3. 08049 | | 1389 | coupon | 3. | 2. 80% | 2. 99681 | | 1571 | coupon | 3. 7 | 2. 80% | 2. 91497 | | 1753 | coupon + Principal | 103. 7 | 2. 80% | 79. 46664 | | | | | | | |

| | | 108. 84 | a) Dirty price: is the present value of all cash flows, therefore equal to 108. 4 b) Accrued interest: is the coupon accrued from the last coupon date until the settlement date. Given a coupon of 3. 7, accrued period of 68 days and total coupon period of 183 days, the accrued interest at settlement date is equal to: [pic] c) the clean price is the difference between the dirty price and the accrued interest: 108. 84-1. 374= 107. 47 Question A. 3 You invest in the following Zero Coupon Bond with one year maturity: | starting date | 30/09/2011 | | maturity date | 30/09/2012 | | day count | 30/360 | nominal value | 100 | If you buy 5 Million € of face value for a yield of 2%, you will spend 4, 901, 961 Euro (check! PV of the 5 million that you will receive at maturity…) a) What is your profit/loss if you sell this security on the same day for a yield of 3%? Calculate also the duration of this security (NB: remember the definition of duration) b) What will be the price of the security on 30/04/2012 if you can see it for a yield of 2%? What is the profit/loss on your investment if you decide to sell it on 30/04/2011 at 2% yield? c) What is the duration of this security on 30/04/2012? NB: you can calculate it changing the yield to maturity by 1% and … ) Solution a) if the security is sold at 3% yield to maturity, its PV is equal to: [pic] In this case the profit would be 4, 854, 369-4, 901, 961=-47, 592. The duration will be equal to the maturity, given that it is a zero coupon bond, i. e equal to 1. The duration can be calculated also dividing the loss/profit from a 1% change in yield to maturity with the invested amount: [pic] b) What will be the price of the security on 30/04/2011 if you can see it for a yield of 2%? What is the profit/loss on your investment if you decide to sell it on 30/04/2011 at 2% yield? [pic]

In this case the profit would be 4, 958, 914-4, 901, 961= 56, 953 c) in order to calculate the duration, we need to know the loss due to an increase in yield to maturity of 1%: [pic] The loss would be equal to 4, 958, 914-4, 938, 797=-20, 117. The duration of this bond is therefore: [pic] Question A. 4 This question has the objective to make sure that you have understood at least the basics of what has been done during the first three lectures. Look at the following equation for pricing bond: given C the coupon (fixed), n the maturity and M the principal amount at maturity: • What are y and P? • Why have they an inverse relation? Under which hypotheses can we have a realized annual return equal to y, if we invest in this bond? Which are the three components of the realized return? Why do we need to measure them? • What is the relation between C, n and the modified duration of this bond? • What risk do we measure with duration? Why it is only an approximation of this risk? Solution a) y is the yield to maturity of this security, or the annual expected yield gained from investing in this security. P is the price of the security, which is equal to present value of all the future cash flows, discounted using y as discount rate. ) y and P have an inverse relation because when a higher yield is applied to discount the future cash flows, their present value is lower, therefore the price of this security is lower. Differently said, if the yield to maturity required by an investor for buying this security is higher, the price of the security must be lower (given fixed future cash flows). c) The realized return gained investing in a fixed income security will be equal to the expected annual return if the security is held to maturity and all the interim cash flows can be reinvested with an annual yield equal to the yield to maturity.

The three components of the realized return are coupon payments, capital gain/loss (i. e. the difference between the purchase price and the sell price) and the interest gained reinvesting the interim cash flows. It is useful to know them because we want to know how much of our return (or expected return) is “ fixed” (coupons) and how much depends on our assumptions on future level of interest rates (capital gain and interest on interest). The more the realized return depends on the second and third components, the higher is the risk due to our hypotheses, i. . the cost of making the wrong assumptions. d) Modified duration can be seen as the average life of our bond, weighted by the cash flows. Higher duration means that we have to “ wait more” before receiving the promised cash flows. This is the case when n is higher and also when C is lower. e) With duration we measure the interest rate risk. In particular duration is the approximate percentage change in the price of the security for a 1% change in the yield to maturity. ————————– [pic]