

The history of integration, its uses, and applications essays example

[Engineering](#), [Aviation](#)



1. Integration is a topic within calculus that has been applied in many engineering problems. Sir Isaac Newton and Gottfried Wilhelm von Leibniz first developed integration concepts, while working independently, in the 17th century (Goldman and Foresta, 2005 p. 4). Leibniz used integral calculus to work on sequences of functions while Newton used it to find the area under a curve and consequently overlook the use of indivisibles (Goldman and Foresta, 2005 p. 5). A separate, but similar in concept discoveries, lay the foundation for the two part fundamental theorem of calculus.

The first part states that, if f is a continuous function on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a)$$

Weisstein.

The second fundamental theorem of calculus holds for f a continuous function on an open interval I and a any point in I , and states that if F is defined by the integral

$$F(x) = \int_a^x f(t) dt,$$

then

$$F'(x) = f(x)$$

at each point in I , where $F'(x)$ is the derivative of $F(x)$ (Weisstein).

2. To find the surface area and volume of the following backpack, we are going to use the first fundamental theorem of calculus to find the surface area and the volume.

First we trace the shape of the backpack resting on the xy plane. This will be our function on the xy plane. Let it be called $h(x)$. The outline of the bag has extreme points $(2, f_x)$ and $(4, g_x)$. $f_x = x^2 + x$ and $g_x = x^2 - 2x$

the functions $f(x)$ and $g(x)$ are the equations of the line from the top and bottom respectively, with respect to the x -axis

The outline of the bag is as shown below

Calculating the surface area will use the formula

$$25 \int f(x)g(x) \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dy dx$$

$$\text{where } h_x, y = 1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = 0$$

$$\left(\frac{\partial z}{\partial y}\right)^2 = 0$$

the two values are each 1 because $z = z$. therefore the surface area will be

$$25 \int_0^2 \int_0^{2-x} \sqrt{1 + 0 + 0} dy dx = 25 \int_0^2 (2-x) dx$$

this equals

$$25 \int_0^2 (2-x) dx = 25 \left[2x - \frac{1}{2}x^2 \right]_0^2 = 25(4 - 2) = 50$$

the surface area is thus 50 square units.

$25 \int f(x)g(x) \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dy dx$

which gives us,

$$25 \int_0^2 \int_0^{2-x} \sqrt{1 + 0 + 0} dy dx = 25 \int_0^2 (2-x) dx$$

i. e

$$25 \int_0^2 (2-x) dx = 25 \left[2x - \frac{1}{2}x^2 \right]_0^2 = 25(4 - 2) = 50$$

$$= 50$$

However, volume cannot be negative. Therefore it is

50 = 50 cubic units.

3. Integration by parts is a method of solving indefinite integrals. It is an integration form of the product rule. The rule is based on the product rule

formula for derivatives. We know that the inverse of a derivative is its integral and vice versa. Therefore,

$$d(uv)dx = vdu + u dv$$

Where, du and dv are the derivatives of u and v respectively.

Integrating both sides results in,

$$d(uv)dx = vdu + u dv$$

$$\text{or simply } d(uv) = vdu + u dv$$

$$uv = \int vdu + \int u dv$$

rewriting this formula, (with the assumption that the functions whose integral we are looking for is u)

$$\int u dv = uv - \int v du$$

This is the formula for integration by parts (Edelstein-Keshet, 2010 p. 126).

$$\int \ln x dx$$

In these example let $u = \ln(x)$ therefore $du = \frac{1}{x} dx$.

$$dv = dx \text{ implying that } v = x$$

$$\int \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x$$

integrating on the right side will yield

$$\int \ln x dx = x \ln x - x + C$$

where C is a constant.

Assuming we had taken $u = dx$ and $dv = \ln(x)$ then we would have $du = 1$ while the answer for dv would be like solving the original question, which is what we are working towards. Therefore, the choice of u and dv is important when using integration by parts to avoid repetition of the question.

References

Edelstein-Keshet, L. (2010). Integral Calculus: Mathematics 103. Vancouver: University of British Columbia.

Goldman, L & Foresta, S. (2005). Principia Mathematica Historallis Integratus.

Retrieved from http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=5&ved=0CFAQFjAE&url=http%3A%2F%2Fwww.math.rutgers.edu%2F~mjraman%2FHist%2520of%2520Integrals.pdf&ei=ZjRqU9bvLlvX7AayqoHYDg&usg=AFQjCNFRBEdbz1OL7GMsCB06kuDrnKyLgg&sig2=0g6dOpdbjw2AbBF7K_SHCw&bvm=bv.66111022,d.ZGU

Weisstein, E. W. Fundamental Theorems of Calculus. Retrieved from

MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/FundamentalTheoremsofCalculus.html>