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is . solving the



If someone wants to run a regression he has to check if the auxiliary variables of the regression are not stationary 1, if not he has to use the first differences of those variables.

It is of great importance for the auxiliary variables to be stationary. In case of non-stationarity, any deviation from equilibrium will not be temporary. This of course is the safe way which has been used in many regressions of time series ever since Granger & Newbold published their paper about the problem of spurious regression. Apparently, that technique cannot be considered flawless. The need of using the levels and not the first differences of the variables "created" the meaning of cointegration.

Cointegration In most of cases, the linear combination of two variables which are  $I(1)$  is also  $I(1)$ . In general, variables with different orders of integration are combined, their combination order of integration equals the largest. So, if for  $i$ , we have variables each integrated of order  $d_i$ , so that the integration order of  $X$  is  $d$ . Solving the above requisition with respect to  $X$ , we have: Where  $u$  The equation can be considered as a new regression where  $u$  is a disturbance term.

This disturbance term has two unwanted properties: first it is not stationary in most of the cases and secondly it is autocorrelated since all the  $u_t$  are Let's consider an example: The sample regression function of the above equation will be written as follows: If we solve the above requisition with respect to  $X$  we have: We have expressed the residuals like a linear combination of our auxiliary variables. In most of the regressions the combination of non stationary variables will itself be non stationary but this is not very

convenient. The perfect case would be the residuals to be stationary. This is the case when the variables are cointegrated. Back in 1987, Engle and Granger proposed the following definition about cointegration. Let  $X_t$  be a vector of variables integrated of order  $d$ . All  $X_t$  are  $I(d)$ . There is at least one vector of coefficients  $\alpha$  such that  $\alpha'X_t$  is stationary. In reality most of the financial variables have one unit root so they are  $I(1)$ . Having this in mind, a set of variables is considered cointegrated if their linear combination is stationary.

It has been observed that many time series may not be stationary, but they may be related in the long run. A cointegration relationship is also a long-term or else equilibrium phenomenon because it is possible that cointegrated variables may seem unrelated in the short run but this is not true for the long run. In this point it is important to distinguish the meaning of spurious relationships with cointegrated ones. The spurious regression problem is appeared when totally unrelated time series may appear to be related using traditional testing procedures. And from the other hand, we face genuine relationships which arise when the time series are cointegrated. Engle and Granger test Engle and Granger in 1987, recommended that « If a set of variables are cointegrated, then there exists a valid error correction representation of the data, and vice versa ».

To put it differently, if two variables are cointegrated there must be some force that will make the equilibrium error to go back to zero. Engle and Granger in 1987, also suggested a two-step model for cointegration analysis. For example, let's say that we have an independent variable  $X_t$  and a dependent one  $Y_t$ .

First of all, it should be estimated the long-run equilibrium equation: We run a OLS regression and we have: We solve the above equation with respect to and we have: In practice a cointegration test is a test which examines if the residuals have not got unit root. To examine this, we run a ADF test on the residuals, but we use the MacKinnon(1991) critical values. If the hypothesis of the existence of cointegration cannot be rejected, the OLS estimator, is said to be super-consistent. This means that for a very big sample it is not necessary to include variables in our model. The only important thing from the above test is the stationarity of the residuals, if they are stationary (no unit root) we can move to the second step. So, we save the residuals from the OLS and we proceed to the second step.

The second step we use the unit root process for the stationarity of the residuals to the next equation: The above equation does not have constant term because of the fact that, the residuals have been calculated with the method of ordinary least squares, so they have zero mean. The test suggested from the Engle Granger is a little bit different from those of the one of Dickey-Fuller. The hypothesis of this test is:  $H_0$  : (no cointegration)  $H_1$  (cointegration) The null hypothesis can be rejected only when ( $\tau$  is the critical value of Engle-Granger table). The Engle-Granger Test can be also used for more than two variables.

The process is alike the one we have described. In conclusion the cointegration process is a way to estimate the long run relation between two or even more variables. Engle and Granger in 1987 proved that if two variables are cointegrated, then they have a long run relation equilibrium, while in short run this may not be true. To check if there is a short run

dis-equilibrium we can use an Error Correction Mechanism (ECM).

The equilibrium error can be used to combine the long run with the short run with the help of ECM.

The equation of this model is: Where:  $\alpha$  : is the equilibrium error  $\beta$  : is the short run coefficient which has to be between 0 and -1.  $\Delta Y$  and  $\Delta X$  : are the first differences of  $Y$  and  $X$  which are not stationary We now can now use ordinary least squares since all the variables are . It is important to point out that long run equilibrium is tested through the p-value of coincidence . If  $\alpha$  is significant then  $Y$  causes  $X$  in the long run. Furthermore, the coefficient measures the speed of adjustment to the long run equilibrium.

The higher this coefficient the faster the return to the equilibrium. 1

Integration is when in a one variable context,  $X$  is  $I(d)$  if its (d-1)th difference is stationary?  $X$  is  $I(0)$  if  $X$  is .