Errors in solving equations reducible to quadratic form education essay

In the General Cambridge Examination Ordinary (G. C. E. O) Level Additional Mathematics syllabus, solving equations reducible into quadratic form plays an important role as it affect almost all mathematics strands in the syllabus: Algebra; Geometry and Trigonometry; Calculus. The significant numbers of topics in Additional Mathematics that requires the concept of identifying and solving equations reducible to quadratic form highlights the importance of this study.

Problem Statement

Students in Singapore secondary schools are expected to acquire and master the skill of algebraic manipulation and solving equations particularly linear equations and quadratic equations by the end of Secondary 2 (Grade 8) (Ministry of Education, 2007). Students who take Additional Mathematics in their Upper Secondary (Grade 9 – Grade 11) are expected to be able to transfer their algebraic knowledge and skills particularly in solving equations and manipulation concepts and use it to solve more complex equations. Complex equations solving in Additional Mathematics involves higher-degree algebraic powers, exponential functions, logarithm functions and trigonometry functions. Over the years, I have observed many students who were unable to solve equations reducible into quadratic form when given different functions as mentioned. This present study is an attempt to examine students' errors in solving equations reducible to quadratic form through written and verbal form as well as provide some reflection on teaching.

Purpose of the Study

This study attempts to identify the types of errors that students make in solving equations reducible to quadratic form. The equations in this study refer to equations involving exponential functions, logarithm functions and trigonometry functions which can be simplified to ax2 + bx + c = 0 (a, b and c are constants and x is the functions mentioned). This study intends to answer the following questions:

What kind of errors do students have when solving equation that is reducible to quadratic form?

Why do these students' make these errors?

How we can avoid such type of errors?

Significance of the Study

In mathematics, equation solving is an important skill in advance algebra topics. The use of equation as a problem solving tool is applicable to other disciplines such as Physics. The error analysis that is being carried out in this study would contribute to an understanding of the cause of students' failure in solving equations and to improve classroom instruction. Error analysis reveals both common and unexpected errors. Recognizing and understanding the source of errors made by students will help teachers to plan instructional activities to avoid some of the common errors if teachers are aware of the nature of their students' misunderstanding. An understanding of students' errors in equation solving will provide a better successful remediation action by teachers.

Conceptual Framework

The following is the conceptual framework of the study. Sources of errors can be found at any stage and the errors will be analysed.

Equation in terms of p, where p are function such as exponential, logarithm or trigonometry

Substitution, let p = x

Identify structural features

Quadratic equation form:

ax2 + bx + c = 0

Interpret solution

Solve for p

Solve for x

LITERATURE REVIEW

In the literature review, a general overview of algebra will be discussed, followed by difficulties in learning algebra and difficulties in solving equations. Next, research of types of errors will be discussed. Lastly a review on understanding mathematics concepts is examined.

2. 1 Algebra

Algebra involves variables; whereas algebraic expressions contain variables, constants and operation signs; whereas algebraic equation contains

algebraic expressions and equal sign. Students are required to understand the concept of variables, the meaning of algebraic term before reducing algebraic expression correctly (Filloy & Rojano, 1989). Thereafter, students need to know that that the structure of an equation is based on equality of algebraic expressions (Kieran, 1981; 1989). Variable plays an important part in advanced therefore I am using Usiskin (1988) conception of school algebra into four conceptions where he emphasized on the role and meaning of variables in each of his conception. One of the important features of variable in this study is viewing algebra as a study of structure (Usiskin, 1998).

2. 2 Studies on Difficulties in Learning Algebra

Many teachers and students will readily agree that algebra is difficult to learn. Many studies have been conducted to identify or explain students' difficulties in algebra are mainly from primary or middle school. The studies on algebra are mainly focused on variables, expressions and solving linear equations (Kuchemann, 1981; Rosnick, 1981; Sleeman, 1984; Booth, 1988; Kieran, 1989; Wagner and Parker, 1993). There are few studies on the difficulties or misconceptions on solving quadratic equations (Vaiyavutjamai & Clements, 2006) in various National Council of Teachers of Mathematics (NCTM) research publications. There is also fewer studies conducted on algebra which are focused on high school or university level (Wagner & Parker, 1993; Vaiyavutjamai & Clements, 2006; Novotná & Hoch, 2008).

2. 3 Difficulties in Equation Solving

Students' difficulties in understanding the structure of algebraic expression will have an impact on solving equations (Kieran, 1981, 1989). Research on equations and solving equations dealt with students' recognition and use of

structure, either implicitly or explicitly (Kieran, 1989). In recognizing the structure of different types of equation, solving it requires different procedures. Hence, students require the correct procedure before they can successfully solve an equation correctly. Ekenstam & Nilsson (1979) study found that the position the unknown as well as the types of solution in the equation affect the performance of the students.

Using concrete models approach to understand the concept of equality to minimize students' difficulties in solving equation was successful in Filloy & Rojano (1989) study. However, this study does not support situation where negative numbers are used. Lima (2007) cited Linchevski and Sfard (1991) research where students made mistakes in solving equation as they misinterpret the techniques used to solve equation as well as lack the meaning of the mathematical symbols.

2. 4 Studies on Error Analysis

Mathematical errors are a common phenomenon in students' learning of mathematics and there is a long history for error analysis in mathematics education (Peng, 2010). Knowledge of the common mathematical errors and misconception of students can provide teachers with an insight into student thinking and a focus for teaching and learning (Chua and Wood, 2005; Kaur, 1989; Ryan and McCrae, 2005; Wong, 2000)

2. 4. 1 Types of Errors

Errors can be classified as procedural and conceptual error

Bagni (2000) concluded in his research that high school students improperly extend simple rules into two types; misconceptions of linear mappings and balance misconception. These misconceptions are operational misconceptions which were caused by an improper over-use of metaphorical projections. Bagni (2000) classified the following examples as misconception of linear mappings, while Wong (2000) classified it as inappropriate use of distributive law:

; sin ;

Example of balance misconception is as follows:

where

Bagni (2000) suggested that to overcome these errors, the role of counterexamples plays an important part to make students aware of incorrect answers and of their conflicting ideas.

Some of the infamous mix-up rules are in solving of algebraic equations using the “ move-over and change-sign”(Wong, 2000) or “ Switching-Addends” error (Kieran, 1989) and “ Redistribution” error (Kieran, 1989). Wong (2000) emphasis that is was no surprise that students often mix up the rules because they do not have relational understanding of what they are doing and their long-term memory is jumbled with numerous rules that look similar.

2. 5 Understanding

Understanding was defined as making connection (Hibert & Carpenter, 1992). Understanding new concept means to build a relationship between the new concept and the old concept. Hibert and Carpenter (1992) suggested different ways to facilitate understanding by going through tasks which requires reflection, communication and working on authentic or real-life problems. This is verified by the study of Brenner et. al (1997) with junior high school students on the concept of functions. Errors are the symptoms of misunderstanding. If the reasons on why students made errors can be well understood, it should be easier to improve their understanding.

2. 5. 1 Conceptual and Procedural Understanding

Conceptual and procedural understanding in mathematics has its place in mathematics. There have been two schools of thought when it comes to teaching and learning of mathematics (Long, 2005). Novotná and Hoch (2008) study was with high school students and university students, where they have difficulties in developing deeper understanding of mathematical notions in their mathematics courses even though, they were high-achieving students in middle-school or high-school. These students have lack of structure sense a term created by Linchevski and Livneh (1999), in other words they do not have conceptual understanding of structure. Kieran (1989, 1992) discussed students' inability to distinguish structural features of equations. Thomas and Tall (1991) indicated the versatility of thought is necessary to switch from an analytical approach to a global one, giving as an example to see 3x + 5 as a common factor in the expression (3x + 5)2 – 2x(3x + 5). These examples indicate that students are required to have

conceptual understanding in learning mathematics concepts when it comes to advance mathematics.

METHODOLOGY

A qualitative approach was chosen for this study, as I want to find out about the mathematical thinking underlying the errors when solving equations. Experimental approach or quantitative approach will not be suitable to seek the information required in the research questions. In order to present a broad picture of the students' errors and understanding, it was necessary to use a written test. Semi-structured interview sessions will be conducted after the analysis of the written test. Interviews session will be the platform where students provide insights into the reasons for errors. Sources of data from the written test, interview sessions and past markers' report will be able to triangulate the source of errors which is required in this study. The sample of the study is secondary Five (Grade 11) students. They were selected for the study as these students would have covered most types of equations as compared to secondary Four (Grade 10) students. The students' knowledge in mathematics could be tested in more situations and would draw out greater variety of errors.

This study was planned to:

examine the error patterns in stages of solving equations reducible to quadratic form,

compare the error patterns of solvers of different abilities in solving equations,

identify the thought process underlying the error patterns.

WORKPLAN

All students are given the instrument containing 7 questions on solving equations. Duration of the test is 40 minutes. The written solution will be marked and errors that are surfaced for each question will be written and categorised. Next, the marks of the test will be tabulated and the students will be categorized to three bands: high ability; medium ability; low ability. The errors from each band will be further analysed. Interview sessions with a few students from each band will held to find out why these errors occur. During the interview session, worked out solution of the students' own working and other students solutions will be used to analyse how the errors occurred. Interview session will be held in the period 7 March – 11 March 2011. In the weeks to come, I will finalize my findings from the interview. I will continue to read more literature on symbols, understanding, errors and equation solving for preparation of the final report.