Mth sl type ii portfolio – fishing rods

Entertainment

Math Summative: Fishing Rods Fishing Rods A fishing rod requires guides for the line so that it does not tangle and so that the line casts easily and efficiently. In this task, you will develop a mathematical model for the placement of line guides on a fishing rod. The Diagram shows a fishing rod with eight guides, plus a guide at the tip of the rod. Leo has a fishing rod with overall length 230 cm. The table shown below gives the distances for each of the line guides from the tip of his fishing rod.

Define suitable variables and discuss parameters/constraints. UsingTechnology, pot the data points on a graph. Using matrix methods or otherwise, find a quadratic function and a cubic function which model this situation. Explain the process you used. On a new set of axes, draw these model functions and the original data points. Comment on any differences. Find a polynomial function which passes through every data point. Explain you choice of function, and discuss its reasonableness. On a new set of axes, draw this model function and the original data points. Comment on any differences.

Using technology, find one other function that fits the data. On a new set of axes, draw this model function and the original data points. Comment on any differences. Which of you functions found above best models this situation? Explain your choice. Use you quadratic model to decide where you could place a ninth guide. Discuss the implications of adding a ninth guide to the rod. Mark has a fishing rod with overall length 300cm. The table shown below gives the distances for each of the line guides from the tip of Mark’s fishing rod.

Guide Number (from tip)

How well does your quadratic model fit this new data? What changes, if any, would need to be made for that model to fit this data? Discuss any limitations to your model. Introduction: Fishing rods use guides to control the line as it is being casted, to ensure an efficient cast, and to restrict the line from tangling. An efficient fishing rod will use multiple, strategically placed guides to maximize its functionality. The placement of these will depend on the number of guides as well as the length of the rod. Companies design mathematical equations to determine the optimal placement of the guides on a rod.

Poor guide placement would likely cause for poor fishing quality, dissatisfied customers and thus a less successful company. Therefore it is essential to ensure the guides are properly placed to maximize fishing efficiency. In this investigation, I will be determining a mathematical model to represent the guide placement of a given fishing rod that has a length of 230cm and given distances for each of the 8 guides from the tip (see data below). Multiple equations will be determined using the given data to provide varying degrees of accuracy. These models can then potentially be used to determine the placement of a 9th guide.

Four models will be used: quadratic function, cubic function, septic function and a quadratic regression function. To begin, suitable variables must be defined and the parameters and constraints must be discussed. Variables: Independent Variable: Let x represent the number of guides beginning from the tip Number of guides is a discrete value. Since the length of the rod is finite (230cm) then the number of guides is known to be finite. Domain = , where n is the finite value that represents the maximum number of guides that would fit on the rod.

Dependent Variable:

Let y represent the distance of each guide from the tip of the rod in centimetres. The distance of each guide is a discrete value. Range = Parameters/Constraints: There are several parameters/constraints that need to be verified before proceeding in the investigation. Naturally, since we are talking about a real life situation, there cannot be a negative number of guides (x) or a negative distance from the tip of the rod (y). All values are positive, and therefore all graphs will only be represented in the first quadrant. The other major constraint that must be identified is the maximum length of the rod, 230cm.

This restricts the y-value as well as the x-value. The variable n represents the finite number of guides that could possibly be placed on the rod. While it is physically possible to place many guides on the rod, a realistic, maximum number of guides that would still be efficient, is approximately 15 guides. Guide Number (from tip) Distance from Tip (cm) 0* 1 2 3 4 5 6 7 8 n** 0 10 23 38 55 74 96 120 149 230 *the guide at the tip of the rod is not counted **n is the finite value that represents the maximum number of guides that would fit on the rod.

Neither of the highlighted values are analyzed in this investigation, they are only here for the purpose of defining the limits of the variables. The first step in this investigation is to graph the points in the table above (excluding highlighted points) to see the shape of the trend that is created as more guides are added to the rod. From this scatter plot of the points, we can see that there is an exponential increase in the distance from the tip of the rod as each subsequent guide is added to the rod. Quadratic Function: The first function that I shall be modeling using the points of data provided is a quadratic function.

The general equation of a quadratic formula is y = ax2 + bx + c. To do this, I will be using three points of data to create three equations that I will solve using matrices and determine the coefficients: a, b and c. The first step in this process is to choose three data points that will be used to represent a broad range of the data. This will be difficult though since there are only three out of the eight points that can be used. Therefore, to improve the accuracy of my quadratic function, I will be solving two systems of equations that use different points and finding their mean. Data Sets Selected: Data Set 1 = {(1, 10), (3, 38), (8, 149)}

Data Set 2 = {(1, 10), (6, 96), (8, 149)} These points were selected for two main reasons. First, by using the x-values 1 and 8 in both sets of data, we will have a broad range of all of the data that is being represented in the final equation after the values of the coefficients are averaged. Second, I used the x values of 3 (in the first set) and 6 (in the second set) to once again allow for a broad representation of the data points in the final quadratic equation. Both of these points (3 and 6) were chosen because they were equal distances apart, 3 being the third data point, and 6 being the third from last data point.

This ensured that the final averaged values for the coefficients would give the best representation of the middle data points without skewing the data. There will be two methods that will be used to solve the system of equations, seen below. Each method will be used for one of the systems being evaluated. Data Set 1 = {(1, 10), (3, 38), (8, 149)} In the first data set, the data points will form separate equations that will be solved using a matrices equation. The first matrix equation will be in the form: Where A = a 3x3 matrix representing the three data points X = a 3x1 matrix for the variables being solved B = a 3x1 matrix for the y-value of the three equations being solved. This matrix equation will be rearranged by multiplying both sides of the equation by the inverse of A: Since A-1*A is equal to the identity matrix (I), which when multiplied by another matrix gives that same matrix (the matrix equivalent of 1), the final matrix equation is: To determine the values of X, we must first find the inverse of matrix A using technology, since it is available and finding the inverse of a 3 by 3 matrix can take an inefficient amount of time.

First let us determine what equations we will be solving and what our matrices will look like. Point: (1, 10) (3, 38) (8, 149) A= The equation is: , X= , B= = Next, by using our GDC, we can determine the inverse of matrix A, and multiply both sides by it. Therefore we have determined that the quadratic equations given the points {(1, 10), (3, 38), (8, 149)} is . Data Set 2 = {(1, 10), (6, 96), (8, 149)} Point: (1, 10) (6, 96) (8, 149) A= , X= , B= The second method that will be used to solve the second system of equations is known as Gauss-Jordan elimination.

This is a process by which an augmented matrix (two matrices that are placed into one divided by a line) goes through a series of simple mathematical operations to solve the equation. On the left side of this augmented matrix (seen below) is the 3x3 matrix A (the new matrix A that was made using data set 2, seen on the previous page), and on the right is matrix B. The goal of the operations is to reduce matrix A to the identity matrix, and by doing so, matrix B will yield the values of matrix X. This is otherwise known as reduced row echelon form. Step by step process of reduction: 1. We begin with the augmented matrix. . Add (-36 * row 1) to row 2 3. Add (-64 * row 1) to row 3 4. Divide row 2 by -30 5. Add (56 * row 2) to row 3 6. Divide row 3 by 7. Add (* row 3) to row 2 8. Add (-1 * row 3) to row 1 9. Add (-1 * row 2) to row 1 After all of the row operations, matrix A has become the identity matrix and matrix B has become the values of matrix X (a, b, c). Therefore we have determined that the quadratic equations given the points {(1, 10), (6, 96), (8, 149)} is . Averaging of the Two Equations The next step in finding our quadratic function is to average out our established a, b, and c values from the two sets data.

Therefore we have finally determined our quadratic function to be: Rounded to 4 sig figs, too maintain precision, while keeping the numbers manageable. Data points using quadratic function Guide Number (from tip) Quadratic values Distance from Tip (cm) Original - Distance from Tip (cm) 1 10 2 22 3 37 4 54 5 74 6 97 7 122 8 149 10 23 38 55 74 96 120 149 New values for the distance from tip were rounded to zero decimal places, to maintain significant figure – the original values used to find the quadratic formula had zero decimal places, so the new ones shouldn’t either.

After finding the y-values given x-values from 1-8 for the quadratic function I was able to compare the new values to the original values (highlighted in green in the table above). We can see that the two values that are the exact same in both data sets is (1, 10) and (8, 149) which is not surprising since those were the two values that were used in both data sets when finding the quadratic function. Another new value that was the same as the original was (5, 74). All other new data sets have an error of approximately ±2cm.

This data shows us that the quadratic function can be used to represent the original data with an approximate error of ±2cm. This function is still not perfect, and a better function could be found to represent the data with a lower error and more matching data points. Cubic Function: The next step in this investigation is to model a cubic function that represents the original data points. The general equation of a cubic function is y = ax3 + bx2 + cx + d. Knowing this, we can take four data points and perform a system of equations to determine the values of the coefficients a, b, c, and d.

The first step is to choose the data points that will be used to model the cubic function. Similarly to modeling the quadratic function, we can only use a limited number of points to represent the data in the function, only in this case it is four out of the eight data points, which means that this function should be more precise than the last. Once again I plan on solving for two sets of data points and finding their mean values to represent the cubic function. This is done to allow for a more broad representation of the data within the cubic function. Data Sets Selected: Data Set 1: {(1, 10), (4, 55), (5, 74), (8, 149)}

Data Set 2: {(1, 10), (3, 38), (6, 96), (8, 149)} Both data sets use the points (1, 10) and (8, 149), the first and last point, so that both data sets produce cubic functions that represent a broad range of the data (from minimum to maximum). The other points selected, were selected as mid range points that would allow for the function to represent this range of the data more accurately. When modeling a cubic function or higher, it is difficult to do so without using technology to do the bulk of the calculation due to large amounts of tedious calculations that would almost guarantee a math error somewhere.

Therefore, the most accurate and fastest way to perform these calculations will be to use a GDC. In both data sets, the reduced row echelon form function on the GDC will be utilized to determine the values of the coefficients of the cubic functions. The process of determining the values of the coefficients of the cubic function using reduced row echelon form is similar to process used for the quadratic function. An x-value matrix A (this time a 4x4 matrix), a variable matrix X (4x1) and a y-value matrix B (4x1) must be determined first. The next step is to augment matrix A and matrix B, with A on the left and B on the right.

This time, instead of doing the row operation ourselves, the GDC will do them, and yield an answer where matrix A will be the identity matrix and matrix B will be the values of the coefficients (or matrix X). Data Set 1: {(1, 10), (4, 55), (5, 74), (8, 149)} (1, 10) (4, 55) (5, 74) (8, 149) A1 = , X1 = , B1 = We begin with the augmented matrix or matrix A1 and matrix B1. Then this matrix is inputted into a GDC and the function “ rref” is selected. After pressing enter, the matrix is reduced into reduced row echelon form. Which yields the values of the coefficients. Data Set 2: {(1, 10), (3, 38), (6, 96), (8, 149)} (1, 10) (3, 38) 6, 96) (8, 149) A2 = , X2 = , B2 = We begin with the augmented matrix of matrix A2 and matrix B2. Then the matrix is inputted into a GDC and the function “ rref” After pressing enter, the matrix is reduced into reduced row echelon form. Which yields the values of the coefficients. The next step is to find the mean of each of the values of the coefficients a, b, c, and d. Therefore we have finally determined our cubic function to be: Once again rounded to 4 significant figures. Updated Data table, including cubic function values. Guide Number (from tip) Quadratic values Distance from Tip (cm) 1 10 2 22 3 37 4 54 5 74 6 97 122 8 149 Cubic values Distance from Tip (cm) Original – Distance from Tip (cm) 10 23 38 54 74 96 121 149 10 23 38 55 74 96 120 149 New values for the distance from tip were rounded to zero decimal places, to maintain significant figure – the original values used to find the quadratic formula had zero decimal places, so the new ones shouldn't either. The y-values of the cubic function can be compared to that original data set values to conclude whether or not it is an accurate function to use to represent the original data points. It appears as though the cubic function has 6 out of 8 data points that are the same.

Those points being, (1, 10), (2, 23), (3, 38), (5, 74), (6, 96), (8, 149). The three data points from the cubic function that did not match only had an error of ±1, indicating that the cubic function would be a good representation of the original data points, but still has some error. We can further analyze these points by comparing the cubic and quadratic function to the original points by graphing them. See next page. By analyzing this graph, we can see that both the quadratic function and the cubic function match the original data points quite well, although they have slight differences.

By comparing values on the data table, we find that the quadratic function only matches 3 of the 8 original data points with an error of ±2, while the cubic function matches 6 of the 8 points with an error of just ±1, which is as small an error possible for precision of the calculation done. Both functions act as adequate representations of the original points, but the major difference is how they begin to differ as the graphs continue. The cubic function is increasing at a faster rate than the quadratic function, and this difference would become quite noticeable over time.

This would mean that if these functions were to be used to determine the distance a 9th guide should be from the tip, the two functions would provide quite different answers, with the cubic functions providing the more accurate one. Polynomial Function: Since it is known that neither the quadratic, nor the cubic function fully satisfy the original data points, then we must model a higher degree polynomial function that will satisfy all of these points. The best way to find a polynomial function that will pass through all of the original points is to use all of the original points when finding it (oppose to just three or four).

If all eight of the points are used and a system of equations is performed using matrices, then a function that satisfies all points will be found. This is a septic function. To find this function, the same procedure followed for the last two functions should be followed, this time using all eight points to create an 8x8 matrix. By then following the same steps to augment the matrix with an 8x1 matrix, we can change the matrix into reduced row echelon form to and find our answer. In this method, since we are using all eight points, the entire data set is being represented in the function and no averaging of the results will be necessary.

The general formula for a septic function is . Data Set: {(1, 10), (2, 23), (3, 38), (4, 55), (5, 74), (6, 96), (7, 120), (8, 149)} (1, 10) (2, 23) (3, 38) (4, 55)

(5, 74) (6, 96) (7, 120) (8, 149) A=, X= , B= , Augment matrix A and matrix B and perform the ‘ rref’ function The answers and values for the coefficients = The final septic function equation is This function that include all the original data points can be seen graphed here below along with the original points. Updated Data table, including septic function values Guide Number (from tip) Quadratic values Distance from Tip (cm) Cubic values Distance from Tip (cm)

Septic values – Distance from Tip (cm) Original – Distance from Tip (cm) 1 10 2 22 3 37 4 54 5 74 6 97 7 122 8 149 10 23 38 54 74 96 121 149 10 23 38 55 74 96 120 149 10 23 38 55 74 96 120 149 New values for the distance from tip were rounded to zero decimal places, to maintain significant figure – the original values used to find the quadratic formula had zero decimal places, so the new ones shouldn’t either. By looking at the graph, as well as the data table (both seen above), we can see that, as expected, all 8 of the septic function data points are identical to that of the original data.

There is less than 1cm of error, which is accounted for due to imprecise (zero decimal places) original measurements. Therefore we now know that the septic function that utilised all of the original data points is the best representation of said data. Other Function: The next goal in this investigation is to find another function that could be used to represent this data. The other method that I will use to find a function that fits the data is quadratic regression. Quadratic regression uses the method of least squares to find a quadratic in the form .

This method is often used in statistics when trying to determine a curve that has the minimal sum of the deviations squared from a given set of data. In simple terms, it finds a function that will disregard any unnecessary noise in collected data results by finding a value that has the smallest amount of deviation from the majority of the data. Quadratic regression is not used to perfectly fit a data set, but to find the best curve that goes through the data set with minimal deviation. This function can be found using a GDC. First you must input the data points into lists, (L1 and L2).

Then you go to the statistic math functions and choose QuadReg. It will know to use the two lists to determine he quadratic function using the method of least squares. Once the calculation has completed, the data seen below (values for the coefficients of the function) will be presented: QuadReg a = 1. 244 b = 8. 458 c = 0. 8392 With this data we can determine that the function is When graphed, this function has the shape seen below: Updated Data table, including septic function values Guide Number (from tip) Quadratic values Distance from Tip (cm) Cubic values Distance from Tip (cm)

By analyzing the graph and values of the quadratic regression function, it is evident that it is a relatively accurate form of modeling the data. Four of the eight points matched that of the original data, with an error of ±1. The most notable difference between the quadratic regression function and the quadratic function previously determined, is the placement within the data f the accurate values. The regression function matched the middle data, while the quadratic function matched the end data. It is interesting to see how two functions in the same form, found using different methods yielded opposite areas of accuracy. Best Match: The function that acts as the best model for this situation is the septic function. It is the only function that satisfies each of the original data points with its equation. Through finding the quadratic, cubic and septic functions, it was discovered that the degree of the polynomial was directly correlated to the function’s accuracy to the data.

Therefore it was no surprise that this function acts as the best fit for this data. The other cause for this septic function having the best correlation to the original data is due to the septic function being established by creating a system of equations using all of the data points. 9th Guide: Using my quadratic model, it can be determined where the optimal placement for a ninth guide would be by substituting ‘ 9’ in for x in the equation . Using my quadrating model, it was found that the optimal placement for a ninth guide on the rod is 179cm from the tip of the rod.

Leo’s fishing rod is 230cm long, yet his eighth guide is only 149cm from the tip of the rod. That means that there is 81cm of the line that is not being guided from the reel to first guide. By adding a ninth guide, that distance will be shortened form 81cm to 51cm. By doing this, it will be less likely for the line to bunch up and become tangled in this 81cm stretch where there is no guide. Another implication of adding another guide would be that the weight distribution of a fish being reeled in would be spread over another guide, which will allow for an easier task of reeling in the fish.

There is even enough space on the rod for a 10th guide at 211cm from the tip of the rod. This guide would once again shorten the excess line further to a point where the excess line between the reel and the first guide is shorter than line between the first and second guide. This could cause problems with reeling and casting efficiency, as that extra guide would cause slowing movement of the line. The benefit would be that once again the weight distribution of fish would be spread over a larger number of guides.

Overall, it would be beneficial to include a ninth guide to Leo’s fishing rod, but anymore will likely hinder its efficiency. Mark’s Fishing Rod: Guide Number (from tip) Distance from Tip (cm) 1 10 2 22 3 34 4 48 5 64 6 81 7 102 8 124 To see how well my quadratic model fits this new data, they must be both plotted on the same graph, seen below. My quadratic model for Leo’s fishing rod correlates with Mark’s fishing rod data for the first few values and then diverges as the number of guides increases by growing at a higher exponential rate.

The difference between Leo and Mark’s eighth guide from the tip of their respective rods is 25cm, yet both men’s first guides start the same distance from the tip of their rods. The quadratic function used to model Leo’s fishing rod does not correlate well with Mark’s fishing rod data. Changes to the model must be made for it to fit this data. The best way to find a model for Mark’s data would be to go through the same steps that we went through to determine the first quadratic formula that model’s Leo’s fishing rod.

By doing so, specific values that better represent Mark’s fishing rod data could be used to establish a better fitting function. The main limitation of my model is that is was designed as a function for Leo’s data specifically. It was created by solving systems of equations that used solely Leo’s fishing rod for data. Consequentially, the quadratic model best represented Leo’s fishing rod, which had a maximum length of 230cm, with differently spaced out guides. There were many differences between Leo and Mark’s fishing rods (such as maximum length and guide spacing) that caused my original quadratic model to not well represent Mark’s data.