

# [Computational structural engineering: past achievements and future challenges](https://assignbuster.com/computational-structural-engineering-past-achievements-and-future-challenges/)

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## Past Achievements and Current Trends

Computational methods are computer-based methods used to numerically solve mathematical models that describe physical phenomena. The purpose of computational modeling is to study the behavior of complex systems by means of computer simulations and it can be used to make predictions of the system's behavior under different conditions, often for cases in which intuitive analytical solutions are not available ( [Nature, 2018](#B36) ). Computational methods have emerged in engineering during the 1960s. Since then, structural engineers have been leaders in technological solutions to engineering analysis and design problems. The evolution of electronic computers together with the tremendous increase of computational power has triggered the continuous development of computational methods. Rapid advances in computer hardware have had a profound effect on various engineering disciplines. The applications are numerous and cover a broad field of engineering branches including civil, mechanical, naval, electrical, aerospace, material, biomolecular, among others. Computational structural engineering has evolved as an insightful blend combining both structural analysis and computer science.

Among all computational methods, the Finite Element Method (FEM) and the Boundary Element Method (BEM) are the most prevalent ones. Both methods exhibit unique characteristics as well as advantages and disadvantages. The FEM, as a simulation tool, enables sophisticated methods of computational mechanics, computer technology and applied mathematics. It has been broadly adopted in scientific research and engineering applications and it can be considered as the most popular method used for structural analysis, tackling linear and nonlinear problems of systems with various geometries, material properties and loads ( [Reddy, 2005](#B40) ). In FEM, the solution domain is subdivided into a finite number of elements. Each element approximately reproduces the behavior of a small region of the body it represents, but continuity between the elements is only enforced in an overall minimum energy sense. The FEM is especially suitable for problems with complex geometries, but the whole-body discretization scheme that is utilized inevitably leads to a large number of finite elements and, thus, increased computational cost. Although Professor R. Clough coined the term “ Finite Element Method” in his pioneer work dated back in 1960 ( [Clough, 1960](#B8) ), the answer to the question “ who invented FEM in everyday use?” is possibly M. Jonathan (Jon) Turner at Boeing who generalized and perfected the Direct Stiffness Method, and convinced his company to commit resources to it, over the period 1952–1964 ( [Felippa, 2017](#B12) ). The FEM obtained its real impetus in the 1960s and 1970s by the developments of pioneers J. H. Argyris, R. W. Clough, H. C. Martin, O. C. Zienkiewicz and their co-workers who evolved the method and applied it to a wide range of structural problems and applications.

In the years since its first use, FEM has grown and developed into a standard in design engineering worldwide. While the applications and technological capabilities may vary between different software programs, the cornerstone principle of the methodology remains the same. Although significant developments have been made in FEM over the past decades, there are still many technical challenges that remain outstanding, while new challenging problems keep emerging with the growth of explorations in science, engineering and technology. New novel principles, techniques, algorithms and methods are continuously being developed to improve the precision, speed, robustness and applicability of FEM.

On the other hand, BEM has proven to be an alternative to FEM computational method which offers different computational potentialities. In BEM the governing differential equations are initially transformed into equivalent integral ones, which in turn are discretized on the boundary of the solution domain. In this respect the dimension of the problem is reduced by one order and the number of unknowns is significantly reduced as well. Additionally, BEM allows the evaluation of the derivatives of the solution at any point of the domain of the problem, whereas it is suitable for the analysis of structures with complex boundaries and geometric peculiarities. In 1903, Fredholm was the first scholar who employed singular boundary integral equations ( [Fredholm, 1903](#B13) ), as a mathematical tool, to calculate the unknown boundary quantities for potential problems. Since at that time closed form solutions could only be derived for simple geometries, the method was practically ignored until the beginning of electronic computers. It was not until the early 1960s when [Jaswon (1963)](#B18) and [Symm (1963)](#B42) used Fredholm's equations to solve some two-dimensional problems of potential theory ( [Katsikadelis, 2016](#B23) ). A special reference should be made to pioneer Prof. Carlos Brebbia, renowned throughout the world as the originator of the BEM, who passed away only recently. He wrote one of the first books in BEM ( [Brebbia, 1980](#B4) ) and organized the first international conference on Recent Advances in BEM, back in 1978.

In the following years the development of the two computational methods was expeditious with numerous applications in engineering practice including: static and dynamic analysis of beams, plates, shells and membranes; linear and nonlinear problems of elasticity; orthotropic, anisotropic, composite and layered materials; plasticity; viscoelasticity; earthquake engineering; continuum mechanics; fracture mechanics and geomechanics; soil-structure engineering; size and shape optimization; to name only a few.

In the past two decades, meshfree or mesh reduction methods have been developed gaining tremendous traction among scientists, researchers and engineers. Trying to alleviate mainly the tedious meshing of the traditional element methods (especially FEM), meshfree methods do not require connection between nodes and the approximation of unknowns in the partial differential equations (PDEs) is constructed based on scattered points, within the problem domain and on its boundaries, without mesh connectivity ( [Chen et al., 2017](#B6) ). The earliest development of a meshfree method was the Smoothed Particle Hydrodynamics (SPH) introduced by [Gingold and Monaghan (1977)](#B14) and [Lucy (1977)](#B30) initially for astrophysical problems, while for solid mechanics the method has been applied by [Libersky et al. (1993)](#B28) . The method exhibited many inaccuracies especially near boundaries and tension instabilities.

Many and various meshfree methods have emerged in the literature over the years, which mainly fall into two major categories based on the problem formulation, weak or strong. All the Galerkin meshfree methods belong in the weak form category (see e. g., [Liu et al., 1995](#B29) ; [Babuška and Melenk, 1997](#B2) ) being confronted with domain integration, and special treatment of the essential boundary conditions. On the other hand, the collocation meshfree methods (see e. g., [Kansa, 1990a](#B20) , [b](#B21) ; [Chi et al., 2013](#B7) ) are classified in the strong form category, suffering from very ill-conditioned systems. For more information on meshfree methods, the interested reader can consult the excellent review paper by [Chen et al. (2017)](#B6) .

Similarly, mesh reduction methods use numerical techniques to reduce the domain mesh size or to reduce the discretization over the boundary of the body. For example, in BEM the presence of body forces generates domain integrals that can be computed by domain discretization. This, however, spoils the pure boundary character of the method. Several BEM-based methods have been reported in the literature which overcome the problem of domain integrals evaluation. Among them the Dual Reciprocity Method (DRM) ( [Partridge et al., 1991](#B39) ) and the Analog Equation Method (AEM) ( [Katsikadelis, 1994](#B22) ) are actually the most competent ones. Both methods can maintain the pure boundary character since discretization and integration are limited only on the boundary.

Other interesting and promising methods have also been proposed and developed lately. Isogeometric analysis (IGA) represents a recently developed technology in computational mechanics that offers the possibility of integrating FEA into conventional NURBS-based CAD design tools ( [Hughes et al., 2005](#B16) ). On the other hand, the extended finite element method (XFEM) is a numerical technique which extends the classical FEM approach by enriching the solution space for solutions to differential equations with discontinuous functions ( [Moës et al., 1999](#B34) ). Based on the generalized FEM and the partition of unity method (PUM), the method was developed to ease difficulties in solving problems with localized features (e. g., discontinuities) that are not efficiently resolved by mesh refinement.

There are numerous software packages available today for the simulation and analysis of structures, either commercial, free to use, or completely open-source. The philosophy of open source software is that it is developed by a community, in a collaborative manner, producing a reliable, high quality software quickly and inexpensively. An excellent such example of open-source FEM software is the “ Open System for Earthquake Engineering Simulation” (OpenSees) ( [Mazzoni et al., 2006](#B31) ). It is an object-oriented software framework created at the National Science Foundation (NSF)-sponsored Pacific Earthquake Engineering (PEER) Center, mostly used for the FE simulation of the response of structures subjected to earthquakes.

## Future Challenges in Computational Methods

Given the development of computational methods and relevant tools during the last decades, very powerful capabilities are available today for the simulation and analysis of structures ( [Bathe, 2003](#B3) ). Nevertheless, there are still many exciting research challenges. Some of the future challenges are summarized below.

### System Identification and Modeling of Physical Systems

The foremost challenge in structural engineering and, in general, computational mechanics is two-fold. First, the actual modeling of complex physical phenomena and the derivation of the equations governing their response; second, the development of the necessary computational tools that can accurately solve the respective equations. To these directions System Identification and Fractional Calculus play a dominant role. System identification is the process of modeling an unknown physical system based on a set of known input–outputs values ( [Sirca and Adeli, 2012](#B41) ). In order to identify the constitutive laws of the material or the degree of nonlinearity of the problem, fractional derivatives of constant or variable order ( [Katsikadelis, 2018](#B24) ) must be employed. Moreover, Variable Order (VO)-Calculus, besides the suitable modeling of actual structures, may model the nonlinear response of constant order differential equations as linear response in a VO-Calculus framework, with all the simplifications that arise from the use of linear operators ( [Coimbra, 2003](#B9) ).

### Multi-Scale Modeling

Numerical analysis and simulation at the nano-scale is a major challenge that will open up a huge field in the future. This can lead to applications in biological engineering with the analysis of proteins and DNA. Ideally one would go from nanostructures up to much larger scale structures. Multiscale modeling refers to modeling in which multiple models at different scales of resolution are used to describe a system ( [Ibrahimbegovic and Papadrakakis, 2010](#B17) ). The development of advanced numerical procedures for multi-scale problems is a major challenge as many phenomena in structural engineering involve multiple scales and multi-scale methods are becoming the current trend in many branches of science providing major challenges in fields like nano-technology, fluid flows, bioengineering, material modeling and others.

### Multi-Physics Simulation

To simulate nature, we need to consider multiple physical models or multiple simultaneous physical phenomena. Some of the most exciting challenges is the application of computational methods to problems of multi-physics nature, such as thermo-mechanical; electromagnetic-mechanical; thermo-chemical; fluid-structure interaction; bio-mechanics engineering; etc. Multi-physics methods can be also combined with multi-scale approaches, further increasing the complexity ( [Cross et al., 2007](#B10) ; [de Borst, 2008](#B11) ).

### Modeling 3D Printing

An admirable goal of 3D printing is the creation of material structures that are optimized to fit a particular structural application while at the same time minimizing the material weight and cost (shape or size optimization). 3D printing is still a new technology, particularly when it comes to the mechanical properties of 3D-printed specimens and structures ( [Killi and Morrison, 2016](#B26) ). 3D-printed parts differ from traditionally manufactured parts because of the weak adhesion of vertical layers, a wide variety of printing parameter variables and the behavior of the outer surface, which performs differently than the interior geometry. 3D printing challenges the ways that FEA methods are traditionally employed to assess the structural adequacy of engineered models with the complex internal geometries of 3D printed parts and components ( [Kazakis et al., 2017](#B25) ). The simulation of 3D printing is a coupled problem, a complex process that may involve not only structural strength but also thermal interactions, phase changes and others.

### Modeling of Uncertainties

During the last decades, the engineering community acknowledged the importance of uncertainties on the performance of structures and engineering systems in general ( [Jensen and Iwan, 1992](#B19) ; [Bulleit, 2008](#B5) ). Uncertainty quantification provides metrics to study the relationship between imprecisely prescribed model inputs and the model's predictions ( [Papadrakakis et al., 2005](#B37) ). However, the explicit quantification of uncertainties is still a major challenge ( [Takewaki, 2015](#B43) ). The development of appropriate methods for uncertainty quantification will receive much attention in the years to come with different computational models such as fuzzy analysis, classical probabilities, probabilistic hazard analysis ( [Mori et al., 2017](#B35) ), methods for uncertainty quantification such as robust design, reliability-based design ( [Lagaros et al., 2007](#B27) ), life-cycle optimal design ( [Mitropoulou and Lagaros, 2016](#B33) ), sensitivity analysis and others. These methods will create new opportunities to meet the long-standing challenge of delivering quantitative predictivity in computational mechanics and engineering.

### High-Performance and Cloud Computing

High precision results for large-scale scientific problems often require tremendous computational power, an investment which is expensive and difficult to access. A solution to this problem can be found in high-performance and cloud computing. Although high-performance computing provides new and interesting opportunities to solve large-scale structural engineering problems ( [Papadrakakis et al., 2011](#B38) ; [Hori et al., 2018](#B15) ), the development of new computational models and algorithms that exploit the unique architecture of these machines still remains a challenge ( [Adeli and Soegiarso, 1998](#B1) ). Cloud Computing is a model for enabling convenient, on-demand network access to a shared pool of configurable computing resources that can be rapidly provisioned and released with minimal management effort ( [Mell and Grance, 2011](#B32) ). An example of a high-performance cloud-based open-source framework is the new SimCenter (Computational Modeling and Simulation Center), a component of NSF-supported NHERI (Natural Hazards Engineering Research Infrastructure). The NHERI is a distributed, multi-user facility that provides the natural hazards engineering research and education community with access to research infrastructure. The goal of SimCenter is to provide the community with access to next generation open-source computational modeling and simulation software tools, allowing multidisciplinary specialists to collaborate on solutions to complex natural hazard engineering problems regardless of their local resources and geographic proximity.

## Conclusions

Structural engineering has endured extraordinary challenges in recent years worldwide. Many of the tasks that a structural engineer used to do on his/her own in the past are now being done by computers. New ideas have evolved, outside the scope of prescriptive design codes and the use of computers is dominant nowadays. Very powerful capabilities are now available for the simulation and analysis of structures, yet there are still many exciting research challenges and the field of computational structural engineering will continue to grow and increasingly contribute to technological development. We strongly believe that we are only at the beginning of the use of computer simulations and we only now begin to understand the extent to which these will influence and enrich the engineering profession and our lives in general.

## Author Contributions

VP and GT: Contributed their thoughts and research experience to the conception and design of the work and drafting of the article.

## Conflict of Interest Statement

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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