

Probability exercise



**ASSIGN
BUSTER**

Practice problems Counting

1. A closet contains 6 percent pairs of shoes. Five shoes are drawn at random. What is the probability that at least one pair of shoes is obtained?

2. At a camera factory, an inspector checks 20 cameras and that three of them need adjustment before they can be shipped. Another employee carelessly mixes the cameras up so that no one knows which is which. Thus, the inspector must recheck the cameras one at a time until he locates all the bad ones.
 - (a) What is the probability that no more than 17 cameras need to be rechecked?
 - b) What is the probability that exactly 17 must be rechecked?

3. We consider permutations of the string " ABACADAFAG". How many permutations are there? How many of them don't have any A next to other A? How many of them have at least two A's next to each other?

4. A monkey is typing random numerical strings of length 7 using the digits 1 through 9 (not 0). Call the digits 1, 2, and 3 " lows", call the digits 4, 5, and 6 " mids" and digits 7, 8, and 9 " highs".
 - (a) How many do event strings can he type?
 - (b) How many of these strings have no mids?
 - (c) How many of these strings have only one high in them? For example, the string 1111199 has two highs in it.
 - (d) What's the probability that a string starts with a low and ends with a high?

- (e) What's the probability that a string starts with a low or ends with a high?
- (f) What's the probability that a string doesn't have at least one of the digits 1 through 9?

5. School of Probability and Statistics (SPS) at IUA University has 13 male Moroccan professors, 8 female Moroccan professors, and 12 non-Moroccan professors. A committee of 9 professors needs to be appointed for a task.

- (a) How many committees can be made?
- (b) What's the probability that the committee contains 2 Moroccan women, 3 Moroccan men, and 4 non-Moroccans?
- (c) What's the probability that the committee contains exactly 4 non-Moroccans?
- (d) What's the probability that the committee contains at least 4 non-Moroccans?
- (e) What's the probability that the committee does not contain any Moroccan men? Conditional Probability, Bayes'.

Theorem

1. Before the distribution of certain statistical software, every fourth compact disk (CD) is tested for accuracy. The testing process consists of running four independent programs and checking the results. The failure rate for the 4 testing programs is, respectively, 0.01, 0.03, 0.02, and 0.01.

- (a) What is the probability that a CD was tested and failed any test?
- (b) Given that a CD was tested, what is the probability that it failed program 2 or 3?

- (c) In a sample of 100, how many CDs would you expect to be rejected?
- (d) Given a CD was defective, what is the probability that it was tested?

2. Does a regional telephone company operate three relay stations at rent locations? During a one-year period, the number of malfunctions reported by each station and the causes is shown below:

Station	Problems with electricity supplied	Computer malfunction	Malfunctioning electrical equipment	Caused by other human errors
A	2	4	5	7
B	1	3	4	7
C	1	2	2	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

3. Police plan to enforce speed limits by using radar traps at 4 do event locations within the city limits. The radar traps at each of the locations L1, L2, L3, and L4 are operated 40%, 30%, 20%, and 30% of the time, and if a person who is speeding on his way to work has probabilities 0.2, 0.1, 0.5 and 0.2, respectively, of passing through these locations, what is the probability that he will receive a speeding ticket? You can assume that the radar traps operate independently of each other.

4. Jar A contains 6 red balls and 6 blue balls. Jar B contains 4 red balls and 16 green balls. A six-sided die is thrown. If the die falls "6", a ball is chosen at random from the jar

- A. Otherwise, a ball is chosen from Jar
- B. If the chosen ball is red, what is the probability that the die fell "6"?

5. The word spelled HUMOR by a person from the United States is spelled HUMOUR by a person from the UK. At a party, two-thirds of the guests are from the United States and one-third from the UK. A randomly chosen guest writes the word, and a letter is chosen at random from the word as written.

- (a) If this letter is a U, what is the probability that the guest is from the UK?
- (b) If the letter is an H, what is the probability that the guest is from the UK?

6. Jar A contains two black balls, jar B contains two white balls, and jar C contains one ball of each color. A jar is chosen at random. A ball is drawn from the chosen jar and replaced; then again a ball is drawn from that jar and replaced. If both drawings result in black balls, what is the probability that a third drawing from the same jar will also yield a black ball?

7. A jar contains 5 red balls and 10 blue balls. A ball is chosen at random and replaced. Then 10 balls of the same color as the chosen ball are added to the jar. A second ball is now chosen at random and seen to be red. What is the probability that the ball was also red?

Mass Functions

1. A dice has 6 sides labeled 1 through 6, and the associated probabilities are a , b , c , d , e , and f respectively. Furthermore, you are told that $P(\{1, 2, 3\}) = P(\{4, 5, 6\})$. This die is tossed once and random variable X is twice the face value that showed up. Answer the following questions about X : 3

- (a) What is the range space of X ?
- (b) Draw the cumulative distribution function of X .
- (c) Write down the probability mass function of X .

2. A jar contains 10 balls, labeled 1 through 10. We will take 3 balls out of the jar. Let B be the random variable that is the highest label among the 3 balls withdrawn. Answer the following questions about B : (a) What is the range space of B ?

- (a) Calculate $p(b)$ for $b = 3, 6, 10$.
- (b) Calculate F
- (c) for $b = 3, 6, 10$.
- (d) Calculate $P(B \geq 8)$.

3. Consider a group of 5 blood donors, A, B, C, D, E, of whom only A and B have type O+. Blood samples will be taken from each donor in random order until an O+ donor is reached. Let the random variable Y be the number of blood samples taken until an O+ individual is reached.

- (a) What is the range space of Y ?
- (b) Write down the probability mass function of Y .

4. A jar contains 15 balls, 10 of the red, and 5 of the blue. Three balls are picked and let R be the random variable that is the number of red balls in these 3 drawn.

- (a) What is the range space of R ?
- (b) Write down the prob. mass func. of R .
- (c) Write down the cumulative distr func. of R .

5. A random variable Z has the following range space and probability mass function:

value	-3	-2	5	0	4	12	20
probability of this value	0.1	0.15	0.05	0.3	0.3	0.1	

- (a) Draw the line graph of this PMF.

- (b) Write down the CDF of Z and draw its graph.
- (c) Calculate $P(Z)$.

6. After all, students have left the classroom, a probability professor notices that 4 copies of the textbook were forgotten behind. At the beginning of the next lecture, the professor distributes the 4 books in a completely random fashion to each of the four students who left the books behind. Let X be the number of students who receive their own book. Determine the pmf of X . Hint: Think of permutations of 4 symbols.

7. Let X be the number of tires on a randomly selected automobile that are rated. Which of the following three $p(x)$ functions is a legitimate pmf for X , and why are the other two not allowed?

$p(x) = \begin{cases} 0.3 & x=0 \\ 0.4 & x=1 \\ 0.4 & x=2 \end{cases}$
 $p(x) = \begin{cases} 0.1 & x=0 \\ 0.2 & x=1 \\ 0.1 & x=2 \\ 0.1 & x=3 \\ 0.2 & x=4 \end{cases}$
 $p(x) = \begin{cases} 0.05 & x=0 \\ 0.1 & x=1 \\ 0.14 & x=2 \\ 0.05 & x=3 \\ 0.3 & x=4 \end{cases}$

8. In our experiment, we pick a random permutation of 1234. Let X be the number of symbols that remained in their original places. For example, if the random permutation is 3214, $X = 2$. Find the pmf of X .

9. In our experiment, we type a random string of length 6 using only the letters A, B, C, D, E, X, Y, Z. Let R be the number of letters that are occurring more than once. So, for example, if the string is "BAYEDA", $R = 1$. If string is "DEBAZY", $R = 0$. If string is "AABAXY", $R = 1$.