

Young's modulus of aluminium beam



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Solid Mechanics Lab Report Experiment to determine the Young's modulus of an aluminium cantilever beam and the uncertainties in its measurement 1.

Abstract: The young's modulus E , is a measure of the stiffness and is therefore one of the most important properties in engineering design. It is a materials ratio between stress and strain: $E = \frac{\text{stress}}{\text{strain}}$ Young's modulus is a unique value for each material and indicates the strength of that material as well as how it will deform when a load is applied. . Introduction: The Young's Modulus can only be derived experimentally, there are no theoretical methods by which the young's Modulus of a material can be calculated therefore in this experiment our aims were: * To calculate the Young's modulus , E of Aluminium from measurement of the end deflection of cantilever beam of aluminium loaded at its free end * To assess the accuracy and precision of this method by comparing the calculated value of E to the known value $E_{al} = 72 \text{ GPa}$ * To measure the deflected shape of the aluminium beam for one loading condition (15N) and to compare this with the theoretical prediction of the beam bending theory for deflection of a cantilever $y_x = \frac{PL^3}{32EI} - \frac{13xL^3}{3}$ 3. Materials and Methods The apparatus shown below was set up and the following equipments were used: * Dial gauge was used to measure deflection of the beam * Magnetic clamp stand (not to affect the bending of the beam) * Solid Aluminium Beam * 15 Weights(1N each) * Clamp to keep it still at one end. * Steel base 4. Results 1.

Load/Deflection behaviour : Equation to calculate the young's Modulus from the slope of deflection vs. load graph $E = \frac{4L^3bd^3(\text{slope})}{3}$ We measured the length (L) the width (d) and the breadth (b) of the beam 5 times and then

calculated the average: Average Length/mm | Average width/mm | Average breadth/mm | 998 | 25.28 | 15.73 | The uncertainties of the slope, length, breadth and width were estimated using the range method: $\Delta x = \frac{x_{\max} - x_{\min}}{N}$

The fractional uncertainty of the length was estimated using: Fractional uncertainty = $\frac{\Delta x}{x}$, but in this experiment we will be using the standard error which is $\frac{\Delta x}{\sqrt{N}}$. The uncertainty of the meter ruler used to measure L is ± 0.5 mm, the uncertainty of the vernier calliper used to measure the width and breadth is ± 0.005 mm. Then using the range method we calculated the uncertainty for L: $\Delta L = \frac{L_{\max} - L_{\min}}{5(\text{number of measurements})} = \pm 0.4$ mm, using the same method we calculated $\Delta d = \pm 0.004$ mm and $\Delta b = \pm 0.01$ mm. We decided to use the highest uncertainties of the measurements, hence obtaining the values $L = (998 \pm 0.5)$ mm, $d = (25.3 \pm 0.01)$ mm, $b = (15.73 \pm 0.005)$ mm. The results we obtained are the following:

Load(N)	D(avg)	Error in D(avg)
0	0	0
1	0.7	0.01
2	1.13	0.01
3	1.68	0.01
4	2.24	0.01
5	2.79	0.01
6	3.41	0.02
7	3.95	0.01
8	4.53	0.003
9	5.09	0.01
10	5.66	0.03
11	6.27	0.02
12	6.8	0.01
13	7.38	0.01
14	7.96	0.01
15	8.53	0.01

Where D_{avg} is the average deflection calculated using 3 different readings. The error was calculated using the range method. The values used for the minimum and maximum trendlines used to calculate the average gradient: Load/N | max | Min | 1 | 0.56 | 0.58 | 15 | 8.54 | 8.52 | Slope = $\frac{0.57 + 0.56712}{2} = 0.569$ Uncertainty in slope = $0.57 - 0.5671 = \pm 0.002$

The following graph was obtained by plotting the average deflection against the load. The average gradient = 0.569 ± 0.002 $E = \frac{4L^3bd^3(\text{slope})}{3} = 69.8$ GPa
 Uncertainty in E using partial differentiation: $\Delta E = \frac{L^2 \text{slope} \cdot bd^3}{3} \Delta L + \frac{4Lb}{3} \Delta b$

$b = 12 \text{ mm}$, $d = 4 \text{ mm}$, slope = 0.20 GPa . Hence $E = (69.8 \pm 0.20) \text{ GPa}$, the actual value of $E_{\text{al}} = 72.6 \text{ GPa}$. Comment: The error bars due to uncertainties in the measured deflection are very small as we can see both from the graph and the uncertainty of the slope. 2. Deflected shape of beam: Deflection measurements were taken from the fixed end of the beam in intervals of 10 cm and the load was kept constant at 15 N.

The following values were obtained and the uncertainties in the deflection were calculated using the range method. Using the formula $y = \frac{PL^2}{2EI}x - \frac{13x^3}{L^3}$, and the formula for the second moment of area, $I = \frac{bd^3}{12}$, where b is the breadth and d is the depth of the cross-section of the beam, our displacement values, the theoretical values for the deflection were calculated. $I = 8.199 \times 10^{-9} \text{ m}^4$. To find the uncertainty in I , we use: $\Delta I = \frac{\partial I}{\partial b} \Delta b + \frac{\partial I}{\partial d} \Delta d$. $b = 0.01 \times 10^{-9} \text{ m}^4$. The uncertainties in our theoretical values were calculated using the partial differentials method: $\Delta y(x) = \frac{\partial y}{\partial L} \Delta L + \frac{\partial y}{\partial x} \Delta x + \frac{\partial y}{\partial E} \Delta E$, where E is the Young's Modulus of aluminium, L is the length of the beam and P is the 15 N load applied.

The table below shows our theoretical values and their uncertainties. Now plotting the experimental and the theoretical values against load on the same graph will allow us to compare the experimental aluminium beam deflection with the theoretical prediction of the beam bending theory. Discussion: We took the average of every single measurement we made in order to reduce the random error in the readings. The uncertainties of our variables were small but as we propagate through the experiment the small errors add up.

Also there will always be a slight difference between the experimental and the theoretical values due to errors in our measurements, uncertainties in our equipment or/and impurities in the material. Conclusion: The known Young's Modulus of Aluminium is $E = 76.2 \text{ GPa}$ and our experimental value was $E = (69.8 \pm 0.2) \text{ GPa}$, hence our method is very accurate but there was however some errors. For our deflection experiment, as we can see from the graph there is a very close fit between the experimental and theoretical values of the deflection of the aluminium beam.

The error bars are very small and therefore have minimal effect on the curves, so we can conclude that the beam bending theory of deflection is an accurate and precise method for calculating the deflection of a cantilever beam. Acknowledgments: The group I was part of when the experiment was performed. References: The equations were obtained from the HSDM Solid Mechanics Laboratories Booklet ; Solid Mechanics lecture notes The uncertainties information and methods from " Experimental Methods" by Les Kirkup