# The relationship between teaching and learning

Whilst researching the relationship between teaching and learning, a clear understanding of Realistic Maths Education (RME) was hard to achieve. The characteristics of RME are giving distinguished name to an existing technique that uses real life situations to mathematical problem. Although it is similar to taking a real-life situation and giving the pupils the mathematical problem of it. This gives short term satisfaction, as the teacher is relating the maths aspect in real-life situation and answering the very question (of mine as well) as to how maths will help me in ‘ real-life’.

The development of what is now known as RME started almost thirty years ago. The foundations for it were laid by Freudenthal and his colleagues, which is the oldest predecessor of the Freudenthal Institute. The present form of RME is mostly determined by Freudenthal’s (1977) view about mathematics.

The teaching and learning of mathematics has been changed as a result of the greater emphasis put on mechanical mathematics consequently Realistic Maths Education (RME) has been developed and is continually in development by the Dutch. When it comes to classroom practice much work is yet to be done (Van den Heuvel-Panhuizen, 1998).

Freudenthal believed that the human activity is never considered fixed. This is very similar to Gardners (1996) (et al Barrington (2004)) idea’s of multiple intelligence which allowed me to understand that there are 8 intelligences which can be divided into three main groups:

visual/spatial,

verbal/linguistic

musical/rhythmic,

logical/mathematical,

bodily/kinesthetic,

interpersonal,

intrapersonal

And naturalistic.

He observed that everybody had a mixture of these eight intelligences and some aspect will be stronger than other, over time and life experiences the skills interchange and vary. This was something that Freudenthal very much believed in and founded idea of mathematics as a human activity, that it can never be considered a fixed and finished theory of mathematics education.

Focusing on the development of pupils knowledge and understand of mathematic was part of the foundation of RME and something that was always taken into consideration when looking at a topic of Mathematics to improve it (Van den Heuvel-Panhuizen, 1998). According to Freudenthal, ' mathematics must be connected to reality, stay close to children and be relevant to society, in order to be of human value'. Instead of seeing mathematics as subject matter that has to be transmitted, Freudenthal emphasised that the idea of mathematics as a human activity. Education should give students the ' guided' opportunity to ' re-invent' mathematics by experimenting for themselves.

In RME, the mathematics is introduced in the context of a well thought out practical problem. In the process of trying to solve the problem the child develops mathematical skills. The teacher uses the method of guided reinvention, by which students are given the chance to try and develop their own informal methods for doing mathematics. Students are encouraged to discuss with fellow students strategies in the classroom, learning from each other’s methods. RME is based on what we know about child development and the development of numeracy, and that it is this body of research that is driving the math education reform (Eade and Dickinson)

The use of context problems is very significant in RME compared to traditional, mechanistic approach to mathematics education, which contains mostly bare sums. If context problems are used in the mechanistic approach, they are mostly used to reiterate learning process. The context problems function only as a field of application. By solving contextual problems the students can apply what was learned earlier in the bare situation.

In RME this is different; Context problems function also as a source for the learning process. Contexts problems and real-life situations are used both to constitute and to apply mathematical concepts. While working on context problems the students can develop mathematical tools and understanding.

In order to fulfil the bridging function between the informal and the formal level of learning, models have to shift from a “ model of” to a “ model for.” It was a detail that Leen Streefland developed in 1985 this crucial mechanism in the growth of understanding. Enabling us to gain greater understanding of how pupils can make that crucial transfer of mathematical understanding from their informal techniques to the formal mathematics (Van den Heuvel-Panhuizen, 1998).

Fractions are my topic focus as a feel that sometime fractions aren't given the focus that is should be given. ' Fractions are without doubt the most problematic area in elementary mathematics education' (Steefland, 2001 pg 6) Fractions cause difficulty to most people because they involve relations between quantities meaning that ½ can take many forms and being able to understand and see that relationship can be very difficult for pupils (Mathematics or mathematizing). Another error that pupils make with fractions is to think that, for example, 1/3 of a cake is smaller than 1/5 because 3 is less than 5. Yet most children readily recognise that a cake shared among three children gives bigger portions than the same cake shared among five children.

When Steefland (1991) talks about how the chapters of fractions in a textbook have evolved he articulates that the textbook became ' transposition of application problems into dressed up arithmetic sums', he goes on to says that the orientation of the whole textbook is set out in a similar manner (pg 9-10). This came from both Bouman and Van Zelm, in addition Diels and Nauta, where they aimed to look at alternative ways of reducing the ' thinking sums' to advocate mental arithmetic more.

Meaning that instead of the textbook being another form of guidance for pupils it was being used as a book with orientation of rules and bare sums (without context). Through reading the history of fraction in textbook it seemed that before the ideas of Simon Stevin in the fifteenth century (that we could simplify the daily practice of fractions by using decimal fractions instead (et al Steefland 1991)) we were indeed looking at fraction in more of a relevant to reality way rather than just the ‘ bare sums’. So this concept we have in textbooks of rules and examples is relatively new it wasn’t until the early nineteenth century that this came about in textbook. Therefore the maths in the realistic sense is what we have all been using for centuries compared to the relatively new idea of simplification to rules and sums.

Observing at schools and looking at their textbooks today I can see that they have been over taken by the original ideas of Simon Stevin. Analysing SMP-Interact C1 where the objective is to ‘ simply fractions’, ‘ put fraction in order of size’ and ‘ add and subtract fractions’, by using the mechanical method of teaching unadorned fractions questions.

Key concepts and ideas of fractions are in coloured boxed attracting the attention to read them immediately like these are important in knowing as a consequence understanding these idea will tell you how to solve certain questions on improver fractions, and adding and subtracting. From looking at the chapter, I can see that there is an expectancy of the Year 7’s to have the knowledge and understand integer counting scales,

Clearly it can be seen here in this textbook that Simon Stevin ideas have really been made a priority in the working of a mathematic textbook. Keijzer & Terwel (2001) state historically we have followed these mechanical rules of mathematics leading pupils having trouble relating their understanding of maths to formal mathematical ideas. Particularly when it comes to learning fractions the pupils are getting confused as to how to relate their ideas of what fractions are in real life to how to answer questions on fractions. This is where the RME can help as it is giving the connection between the pupils mathematical idea to how to answer questions or sums on fractions (formally)

When looking at RME in textbook form it follows the idea of Freudenthal (1973) (et al Keijzer & Terwel, 2001-pg 54) that making that mathematical journey leading to formal mathematical ideas comes from ‘ a series of well chosen examples’. The significance of this being that the example and ‘ sums’ that we give to a pupil need to be well thought out to guide the pupils to an idea of learning maths instead of giving them the formal ruling first.

Over a course of up to 2 decades Streefland help develop a new curriculum in the Netherlands on fractions incorporating the practices of RME (et al Keijzer & Terwel, 2001-pg 55). He thought the main theme of fraction activities ought to be the ‘ fair-sharing’ and how sharing between things will give pupils an understanding of fractions. He looked at fraction language at first to give the pupils a confidence in using fraction language by thinking of fair-sharing being pizza’s shared between some people. This then lead on to comparative fractions, by giving another activity which leads the pupils understanding that sharing three pizza’s with four people is the same as sharing 6 pizza’s with 8 people. These are activities which I have seen in the ‘ Mathematics in context- Some of the parts’ textbook.

In 1991, The University of Wisconsin, in collaboration with the Freudenthal Institute, started to develop a middle-school curriculum based on RME. This curriculum is known as ‘ Maths in Context’ and has now been adopted by numerous schools in the US and currently being trialled in the UK.

Talk about the chapter on fraction how the progression is made from informal mathematical ideas and techniques to the formal mathematics.

Also during an investigation on division in a fractional situation I did ask the question of sharing 3 sandwiches between 4 people, to a bright 11 year old who was just about to take her year 6 SAT’s. Here what she did:

The question I asked the individual was to share 3 sandwiches with 4 people, I presented an image of three sandwiches for them to use if they wanted to.

She struggle in answering this question but she did eventually reach an answer. First she saw that she need to split each sandwich into 4 parts and so had 24 pieces which she could then shared between the four people. She found it in fraction form ‘¼ + ¼ + ¼ = ¾’ but still she was not satisfied with her answer. She then broke the sandwiches into 1/8 and then when into another page to make pictures of a cake split into 8 equal parts which then helped her to understand that each person would get 6/8 of the sandwich. (See appendix A)

Now looking at her response I can clearly see that how Streefland looks into fair sharing as a way of trying to get pupils to understand equivalent fractions. Here the responder wasn’t satisfied in the answer just being ‘¾’, but to give herself a better understanding of what ¾ is, she used her knowledge of equivalent fraction and found that by saying 6/8 she was able to confidently understand what the answer and question meant on how to share 3 sandwiches with 4 people.

When looking through Keijzer & Terwel (2001), ‘ Audrey’s Acquisition Of Fractions: A Case Study Into The Learning Of Formal Mathematics’ I found it very informative to the techniques used to enable the pupils understanding of fractions. They talk about how instead of using the diagrams of cakes (as circles) to represent fractions, they found it is more informative to use rectangular cakes. This gives the pupils an improved visual of being able to compare fraction sizes and to be able to ‘ reflect’ on their work. The rectangular cake is very similar to the number this is argued by Connell and Peck (1993) (et al Keijzer & Terwel, 2001). I can see visualise how the rectangular cake is the informal mathematic which will then lead the pupils onto the formal maths which will be where the pupils will be able to use number lines as an instrument to show fractions.

Another idea that I really liked, both Streefland (1991) and Keijzer & Terwel (2001) realised that in order for pupils to understand fractions they need to have the understanding of ‘ number sense’ with fractions.

‘…teaching strategy……number sense is developed

(i) a language of fractions

(ii) developing the number line for fractions,

(iii) comparing fractions,

(iv) learning formal fractions.’

In this quote I am looking at the comparing of fractions, where the equivalent fractions are the key to formal reasoning with fractions. Meaning that if the pupils do not understand the concept that the ratio fraction is the same for something like ½, 2/4, 3/6…etc then it would be hard for them to move onto the formal mathematics. Keijzer & Terwel (2001) used the example of vertical number line houses as fractions, where the fractions lived in each floor and the lift connected the different floors (see image 1). If the building was 3 floors high then it had 3 stops (1/3, 2/3 and 3/3) if it was a 4 floor building then it would have four floors (1/4, 2/4, 3/4 and 4/4). The pupil would fill in as much information as possible on the fraction lift. Then in a discussion they will be able to make judgement on how it was easier to say ½ rather than 2/4 or discuss other point too. This exercise will give a better visualisation on how different fractions can fit in to the same position on the number line.

This is an example of a different way of making pupils understand where the positions are on a number line. It was taken for the new version of the Maths in Context – Fraction Time, here the pupils would be provided with a table and asked to fill in the fractions in the grid (see image 2). I would then go into to ask pupils to find patterns and colour code any similar fraction that they can see on the grid. Once colour coded the pupils will be able to see that some fractions do have the same position on the number line. Although the number line is now very clear in this exercise it will give the pupils a chance to spot pattern. Spotting patterns is an informal strategy that pupils do like to do and help the pupils lead on to the more formal aspect of comparative fractions. But this table can be used as a reference for the pupils to come back to in future lessons on fractions or even on division and decimal numbers.

I want to definitely look at the two exercises that I took from Keijzer & Terwel and Maths in Context and incorporate them in my lessons of fractions as I feel that I have read up on them but never tried them in a classroom. I am anticipating that the pupils will enjoy the fraction with the grid but I need to make sure that it is well planned and thought has been given to how to present it to students. In the lift building I need to again make sure that I comprehend the activity and how to bring in this idea in the class. What I like about the journals that I have looked at is I can take the exercises which they have presenting taking some which are appropriate to my lesson and then I will be able to compare the results of what I found with what the researcher found.

I find that RME does give the freedom of being able to learn different ways to learn formal maths it does give pupils that independence. However when in ‘ mechanical’ lesson where the teacher provides different ways to solve a question on ‘ adding fractions’ does this not insinuate the same conception. Now looking back after researching I find that to a certain point is does mean the same thing but with RME your not standing at the front of the class and tell the pupils that you can solve it this many ways. You are incorporating and respecting the pupil’s way of resolving a maths problem. When they don’t have an existing technique your using you knowledge of what they know and your ‘ stretching it’ to new territory very slowly and the pupils in most cases don’t actually realise that they are moving on and gain a deeper understanding of fractions. I would use RME in giving pupils an understanding of fraction and generally as well but I will always have a awareness of the procedures in mechanical techniques as well and use then where appropriate when I am planning my lessons on fractions.