

# Key concepts of calculus

[Science](#), [Mathematics](#)



Key Concepts of Calculus is the mathematical way of writing that a function of  $x$  approaches a value  $L$  when  $x$  approaches a value  $a$ . For example, if , we can say that which is apparent from the table below

$x$

$f(x)$

$x$

$f(x)$

9

0.111111111

10.1

0.099009901

9.9

0.101010101

10.01

0.0999001

9.99

0.1001001

10.001

0.099990001

9.999

0.100010001

10.0001

0.099999

9.9999

0.100001

10. 0001

0. 099999

9. 99999

0. 1000001

10. 00001

0. 0999999

9. 999999

0. 10000001

10. 000001

0. 09999999

9. 9999999

0. 100000001

10. 0000001

0. 099999999

9. 999999999

0. 1

10. 00000001

0. 1

10

0. 1

10

0. 1

The closer  $x$  becomes to 10, the closer the  $f(x)$  becomes to 0. 1 and here this is bidirectional

Yes it is possible for this to be true in case as is evident from where although

and is undefined.

As for , these functions are called piecewise functions and their limit does not exist as it is not one number  $L$  and they have one-sided limits.

2) means that as the value of  $x$  is reduced and brought closer to a number  $a$ , the value of  $f(x)$  shoots up exponentially. This could be a one-sided limit. For example,

Here the limit does not exist if it is a one-sided limit. Here only the positive half is shown though.

For , a value  $L$  exists such that when  $x$  is allowed to grow unboundedly,  $f(x)$  settles or attempts to settle at  $L$ . For example,

In both of these cases, there are values of  $f(x)$  that are being approached but never settled upon which means that these are its asymptotes. Their limit exists, although some books consider a limit at infinity as non-existent, however, this limit is never reached.

3) For a function to be continuous at a value  $a$ , it means that not only does it equal the function for a specific input value, it is also the value the function approaches both from right and left directions and this limit exists. The three requirements are that,

Condition

Exception

exists

$f(x)$

$f(x)$

4) , is the limit that is reached when the ratio of the difference of the value of  $f(x)$  and that of values of  $x$  is taken relative to a point  $a$ . It is basically the

instantaneous rate of change of  $f(x)$  with respect to  $x$ . Graphically, this is analogous to the formula for gradient that we studied in earlier classes that is  $\frac{dy}{dx}$ . This is basically the gradient of the tangent line to the curve traced by the function.

A function is not differentiable at a point when,

It is discontinuous at that point.

not differentiable at  $x = a$ .

It has a vertical asymptote at that point

not differentiable at  $x = a$

There is a hole at that point

not differentiable at  $x = a$

5. An absolute maximum value is one which is greater than all other values of  $f(x)$  for all values of  $x$ . This means there is an absolute maximum at  $x = a$  if for all values of  $x$ ,  $f(a) \geq f(x)$ . A local maximum is one which is greatest value of  $f(x)$  within a given range of  $x$ . This means

Absolute maximum at  $x = 2$

Local maximum at  $x = 98$  for  $x > 2$

6. is basically a summation of products of countless small rectangles which form a shape on a graph. Here, is analogous to the area of a rectangle where  $A = l \times b$ . The only difference is that as  $f(x)$  is traced along the graph, its height changes. The number  $n$  represents the total number of rectangles made and since it approaches infinity, it means that there width grows shorter and shorter and the whole sum effectively gives the area under the curve with minimum shoot over.

The main difference between definite and indefinite integral in this context is

that the former has a defined domain for which this area is being calculated while the latter has domain equivalent to all real numbers. Definite integral represents area bounded by x axis, the curve and the two vertical lines at limits of domain. Indefinite integral may or may not be bounded from right or left and also, the height of curve is also unknown. So the height of rectangles is also unknown.

7. For the definite integral, bounded below by and above by , of a function, the derivative is the value of function at and the limit is the proof of its continuity. This is true regardless of the value of a.