

Math and music

[Art & Culture](#), [Music](#)



There are some obvious similarities between music and math. A musician might not think about them consciously, but knows innately. Scales and intervals are essentially expressions of mathematical relationships. Grasping the importance of math to music means going beyond the conception that math is more than just numbers.

Musicians at the highest level know that math is, in fact, fundamental to music. Musicians and mathematicians don't often think of themselves in the same terms, but they are actually practitioners of the same art. Music can be studied, created and expressed through the principles of physics and geometry.

Origins

The language of music is heavily laden with numerical jargon. Every young player learns to assign numbers to scale notes and intervals. A musician asked to play a minor seventh interval can visualize this in mathematical form. They think of the scale, then determine its' relationship to what they were asked to play. For most musicians, this kind of math is an unconscious process. It becomes second nature.

Math should not be thought of as a static set of lifeless numbers. In reality, math is a dynamic and creative process of discovering relationships and evaluating their meanings. Music shares the same elements of the problem solving process.

The ancient Greek mathematician Pythagoras was fascinated with music and its expression of mathematical relationships. The beauty of music, for him, was the same as that a mathematician experiences in the process of

discovery (Loy, 2006). He saw the differences in notes in terms of ratios. For example, if C is the tonic D would be $\frac{9}{8}$ of C. The ratio is describing the difference in the number of vibrations. Each note has its own ratio, relative to tonic. The ratio is multiplied for notes in higher octaves, and divided for notes in lower octaves. Generally speaking, the frequency of a note within a scale can be calculated by multiplying the frequency of the tonic note times the frequency ratio for the selected interval (Reid, 2007).

The concept of the octave was described by ancient mathematicians. They discovered that some notes, although different, could be almost imperceptible to the human ear. Correspondingly, they found that there is a mathematical relationship between the same notes in different octaves. The relationships Pythagoras and others illustrated between math and music are fundamental to the development of both fields. In an age when music can be created with computers, the mathematical elements of music have become even more important.

Foundations

Math and music both use symbols and employ conceptual frameworks. As in math, some elements of music are undefined or vague. Yet, the influence of math is apparent in ways musicians may not even consciously realize.

Harvey Reid cites one such example:

The frets of a guitar are actually placed according to the 12th root of 2, and 12 frets go halfway up the neck, to the octave, which is halfway between the ends of the strings. (2007)

A note is the most basic element of music. What is a note? In technical terms, a note is a specific frequency of vibrations. A good example of this can be seen on an electronic tuning meter. The meter does not “hear” notes, per se. It counts the number of vibrations the player is creating and matches it to the appropriate frequency for the note being tuned. The actions the player takes to come into compliance with the tuner are essentially part of a mathematical problem-solving process. This illustrates the “physics” of music.

The time signature itself is a mathematical formula related to number theory. The formula is $n/2m$, where n and m are positive whole numbers (Benson, 2006). More complex time signatures lead to more complicated mathematical relationships. This complexity can be heard, and felt, by the listener.

Scales are expressions of mathematical ratios. Broadly classified, scales may be practice-based, just intoned or tempered (Reid, 2007). All are based on mathematical theories. The intervals present in the scales are expressions of mathematical ratios.

The process of writing music is mathematical at its heart. It consists of constructing many mathematical relationships that, in some way, still relate to each other. We know what intervals to avoid or, in other words, what frequencies clash in an unpleasing way.

Some music takes on geometric properties. A piece in which a theme is repeated and overlapped continually can be described geometrically. The

theme is often stretched, overlapped, changed slightly and moved to different keys. All of these are mathematical actions that could be visualized graphically.

Technology, the future and Conclusion

In the computer age, musicians have a vast array of tools to create new and interesting music. Computers themselves are mathematical machines. Each command is a mathematical problem for the computer to solve. In machines that have vast mathematical capabilities, musicians can explore notational relationships in ways they never thought possible before. There are limitations to the human ear, but technology can use mathematic principles to explore those limitations.

The longer a person studies music, the more they become aware of its mathematical roots. Similarly, the longer a person studies mathematics, the more aware they become music is an expression of mathematical concepts. As technology advances, the potential exists for the each field to increase the knowledge of the other.

Most musicians innately sense the connection with math. The connections go far beyond the obvious, however. It is a linkage that can be traced all the way back to common origins.

Sources

- Benson, David L. (2006). Music: a Mathematical Offering. New York: Cambridge University Press.

- Loy, Gareth. (2006). Musimathics: The Mathematical Foundations of Music Vol. 1. Boston: The MIT Press.
- Reid, Harvey. (2007). “ Of Mathematics and Music”. Accessed 3/22/2007 from:
- <http://www.woodpecker.com/writing/essays/math+music.html>