Good example of mathematics paper report

Sociology, Population

Paper 1

First order differential equations have been very useful in analyzing important properties of a system, such as stability. Literature has seen many works which have explained very specific examples of systems whose dynamics are described by a set of linear first order differential equations. However, there is little evidence, or rather no holistic explanation to prove the usefulness of simple non-linear differential equations in achieving the same. The aim of the paper studied is to bring to light this usefulness, introduce some practical applications of the same, and to examine the behavioral spectrum of such systems. Further, the paper also addresses some pertinent mathematical questions such as using a probabilistic model to describe seemingly random processes.

With this base, a more complicated model is developed for analysis. The final model achieved possesses quite a few practical problems. This includes a contradiction between the inherent mathematical behavior of the system and the inference suggested by numerical simulations. However, there is no doubt that the fundamental idea discussed may well have many applications soon. The paper implored the education system to consider such possibilities and provide students with a strong mathematical base in this area (May, 1976).

Paper 2

Similar in underlying idea to the work discussed above, this paper deals specifically with the modeling of ecological systems. It first shows how difference equations governing a system move from a stable point to stable cycles having periods of 2, 4, 8, and so on, finally leading to apparent chaos. This is achieved through a series of bifurcations as illustrated in an example. Next, the paper covers the implications of such a phenomenon, and reviews many similar studies involving bifurcation phenomena.

The basic difference equation governing the ecological system is Nt+1= FNt, where the X used in Paper 1, which is a continuously varying population, is replaced by a stroboscopic still of the population every year, i. e. a discrete variation. There are many similar equations that are homogenous and mostly possess a critical point. Graphical analysis shows that a fixed point of F (N) = N may or may not have local stability, depending on the Eigen value of N*.

A one parameter model-1 governed by Xt+1= exp [r1- Xt] illustrates the consequences of change in parameters in F. For this model, X* = 1 and λ ® = 1 – r denote the fixed point and the slope at X* respectively. From this, it is evident that the condition 0 < r < 2 must be met for stable equilibrium. For r > 2, the stable point varies, and may change to a repellor.

A deeper analysis of this model shows what exactly happens in the bifurcation process; stable cycles with periods of 2n can be observed as the hump of F steepens. Such a phenomenon may give rise to random behavior and ultimate chaos. In terms of population, this means a series of outbreaks leading to crashes (May & Oster, 1976).

References

May, R. M. (1976). Simple mathematical models with very complicated dynamics. Nature, 261(5560), 459-467.

May, R. M., & Oster, G. F. (1976). Bifurcations and dynamic complexity in simple ecological models. American Naturalist, 573-599.