Vector spaces essay example

[Sociology](), [Identity]()

A]

In order to define and understand a vector space, it is important to first define and understand a few other related terms. To begin with, a vector is a directed line segment, and its extension to n-dimensions leads to a vector space. The addition operation on two ordered pairs of a set is the ordered pair containing the sum of the respective elements. This is true for ‘ n’ number of ordered pairs as well (n-dimensional vector space). Similarly, the scalar product operation on an ordered pair of a set is the ordered pair containing the product of the scalar with the respective elements (“ Vector Spaces”, 2013).

A field F is defined as a set, along with the addition (+) and scalar multiplication (.), such that the following properties are satisfied:

1) Closure Property: For every x, y that belong to F, the values of both (x + y) and (x . y) also belong to F.

2) Commutative Property: This condition requires that for every x, y belonging to F,

x + y = y + x; and

x . y = y . x

3) Associative Property: This requires that for every x, y, z that belongs to F,

x + (y + z) = (x + y) + z; and

x . (y . z) = (x . y) . z

4) Distributive property: This constitutes the following rules which need to be satisfied: For all x, y, and z belonging to F,

x . (y + z) = (x . y) + (x . z)

(x + y) . z = (x . z) + (y . z)

5) The existence of an additive identity: For every x that belongs to F, there exists a number ‘ p’ called the additive identity, such that x + p = p + x = x (For a set of Real numbers, the additive identity is 0)

6) The existence of a multiplicative identity: For every x that belongs to F, there exists a number ‘ p’ called the multiplicative identity, such that x . p = p . x = x (For a set of Real numbers, the multiplicative identity is 1)

7) The existence of both additive and multiplicative inverses: These require the following to be satisfied for every x belonging to F:

Existence of a number ‘ p’ such that x + p = p + x = 0 -> where p is the additive inverse

Existence of a number ‘ p’ such that x . p = p . x = 1 -> where p is the multiplicative inverse

8) For every x belongs to F, the values of both c. x must also belong to F, where c is a scalar multiplier.

A vector space is defined as a set ‘ V’ over a field F on which the addition (+) and scalar multiplication (.) properties are defined, and satisfies the following properties:

1) Closure

2) Associativity

3) Commutativity

4) Existence of identity elements

5) Existence of inverse elements

6) Satisfies unitary law according to which 1 . v = v (“ Vector Spaces”, 2013).

In the above conditions, the properties mentioned are as explained in the beginning – in the definition of a field.

B]

A vector space R2 has two dimensions, that is, it consists of ordered pairs. Mathematically, R2 is denoted as {(a, b) | a, b ∈ R} (“ Vector Spaces”, 2013). Since all the elements of the set ∈ R, the required properties mentioned above are satisfied. Hence the set R2 is a vector space by definition. For example,

For every a, b ∈ R, a + b also ∈ R, thus satisfying closure property.

For every a, b ∈ R, a + b = b + a, satisfying commutative property.

The number zero satisfies the definition of additive identity.

-a ∈ R and 1/a ∈ R, and satisfy the definitions of additive and multiplicative inverses respectively, for an element a ∈ R

For every element a ∈ R, (a . 1) = a, and (a + 0) = a

C]

Let a = (1, 1), and b = (3, 2). For a and b to span R2, there should be a vector p = (p1, p2) ∈ R2 such that there exist r1 and r2 which satisfy the following:

p = (r1. a) + (r2 . b) => p1 = r1 + 3r2; p2 = r1 + 2 r2 (“ Spanning Set”, 2013)

The coefficient matrix then = [1312.] The determinant of this matrix = -1 which is non-zero. Hence the matrix is invertible, and R2 is spanned by a and b.

D]

A subspace is defined as a vector space within a vector space with the same operations. In other words all the elements that belong to a subspace also belong to the vector space which contains the subspace. This is like a subset of a set.

A trivial subspace of R2 is either the 0 vector, or R2 itself, since 0 ∈ R2. They readily fit the definition of a subspace. However, to find a non-trivial subspace S, the following need to be satisfied:

1) 0 ∈ S

2) S is closed under vector addition

3) S is closed under scalar products

4) S is closed under linear combinations

Here " closed" refers to satisfying the closure property which was explained in the beginning.

References

Vector Spaces. (2013). Retrieved October 7, 2013, from http://www. math. ku. edu/~mandal/math290/m290NotesChFour. pdf

Spanning Set. (2013). Retrieved October 7, 2013, from http://www. math. tamu. edu/~yvorobet/MATH304-2011C/Lect2-03web. pdf