Free essay on invertible matrix proofs

Sociology, Identity

Abstract

This paper strives to prove different invertible matrices theorems.

The following are some of the theorems that the paper will justify

- A is an invertible matrix
- A is row equivalent to the n × n identity matrix
- A has n pivot positions
- The equation Ax= 0 has only the trivial solution
- The equation Ax= b has at least one solution for each b in RN.
- The columns of A spans RN
- The linear transformation X→Ax maps RN onto RN.
- There is an n × n matrix C such that AC= I.
- There is an n × n matrix D such that AD= I.
- The columns of A form a basis of RN.

Keyword: a↔b this means a implies b.

Invertible Matrix Theorems

- Definition of Logically equivalent: Two statements are said to be logically equivalent if and only if they are true in precisely the same situation. Mathematically A and B are logically equivalent if one can be proved using the other that is in notation A ⇔ B (Epp, 2011, p. 30).
- Provide an interpretation for the given statement: The n × n matrix A is invertible: n × n matrix A is said to be invertible if there exist a matrix B with the property that AB= BA= I where I is the identity matrix.

Also the” matrix is of order n × n that is there are n number of rows and n number of columns or it is a square matrix of order n. Invertible implies that an inverse exists of the matrix or the value of the determinant of the matrix is a non-zero number" states Kyle.

We can explain this by an example

A= 2 5

1 3where n= 2

A-1= 1/| A|

| A|= (2*3)-(5*1) = 6-5= 1

Hence there exist an inverse A-1 of A

- Justify that the ten statements are logically equivalent to the statement:

The n x n matrix A is invertible.

- Two matrices are said to be row equivalent if and only if they have the same row space that is R (A) = R (B). The row space of the In is the Rn and the row of the An × n are independent, so we can clearly see that R(An × n) = An × n thus a↔b

- Matrix An × n and the identity matrix In is augmented together as (An × n In) and some row operations are done on the matrix to reduce An × n into identity matrix and since An × n is invertible, the final matrix will be (InA-1). since this operation is justified, An × n must have n pivot position hence a↔c.

- If A-1 exist, we have a Ax= A0 implying x= 0 and these is the only trivial solution hence a↔d

- If Ax= b has one solution for every b in Rn, then the matrix A must be invertible by the invertible matrix theorem. That is Ax= Ab implies x= Ab. Thus the equation Ax = b has at least one solution for each b in Rn hence a↔e.

- If A is invertible, then A-transposed is invertible. Therefore, the rows of A-transposed span Rn, so the columns of A span Rn. We can also say that if the columns of A spans Rn the same as saying that Ax= b has a solution for ever b in Rn, but if Ax= 0 has only the trivial solution, then there are no free variables, so every column of A has a pivot, so Ax= b can never have a pivot in the augmented column. Thus Ax= b has a solution no matter what b namely x= A-1 b hence the column of A span Rn that is a↔f (Zhang, 2011, p. 97)

Invertible Matrix Theorems

- (e), (f), and (g) are equivalent for any matrix, (For a particular A, all statements must be all true or all false. C: The columns of A span RM Let T: RN--> RM be a linear transformation and let A be the standard matrix for T. A: T maps RN onto RM if and only if the columns of A span RM). Thus, (f) implies (g) because they are linked to (e) which is linked to (a).
- If A is an invertible matrix then there is an n × n matrix C such that CA= I: C is the inverse of A and a matrix multiplied by it's inverse is the identity. a↔h
- If an n × n matrix A is invertible, then the columns of AT are linearly independent, that is matrix A must have both rows and columns that are independent for it to be invertible. If A is an invertible matrix then there exists an n × n matrix D such that DA= I. If D equals the inverse of A, then DA will equal the identity. a↔i
- We have already shown that columns of A are independent and they forma span of Rn. Therefore they form a basis for Rn. Thus a ⇔ j.

References

Epp, S. S. (2011). Discrete mathematics with applications. Boston, MA: Brooks/Cole.

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Zhang, F. (2011). Matrix theory: Basic results and techniques. New York [etc..: Springer.