

# [Laplace transforms](https://assignbuster.com/laplace-transforms/)

Laplace Transforms – Motivation

### convenience

– differential eqns become algebraic eqns.

– easy to handle time delays

– frequency response analysis to determine how the system responds to oscillating inputs

### Block Diagram Algebra

### – doing math with pictures

### – arithmetic for manipulating dynamic components using boxes and arrows

### Laplace Transform – Review

### Given a function f(t)

Notes –

### f(t) – defined for t from 0 to infinity

### f(t) – suitably “ well-behaved”

– piecewise continuous, integrable

Linearity of Laplace Transforms

### the Laplace transform is a linear operation

### we will use Laplace transforms to analyze linear dynamic systems

### if our models aren’t linear, then we will linearize

Useful Laplace Transforms for Process Control

### We need a small library of Laplace transforms for

– differentiation

– step input

– pulse/impulse functions

– exponentials

– oscillating functions

because these are common functions that we will encounter in our equations

Let’s think about a simple linear differential equation example:

with V and F as constants

Library of Useful Transforms

### differentiation

– initial conditions disappear if we use deviation variables that are zero at an in initial steady state

### unit step function (Heaviside fn.)

Library of Transforms

### exponential

– exponentials appear in solutions of differential equations

» a provides information about the speed of the response when the input changes. If a is a large negative number, the exponential decays to zero quickly

» What happens if a is positive?

– After we have done some algebra to find a solution to our ODEs in the Laplace domain, we must invert the Laplace transform if we want to get a solution in the time domain. We sometimes use partial fraction expansion to express the Laplace expressions in a form that can be easily inverted.

CSTR Example – Transform Model (in deviation variables) using our library of transforms, the Laplace transform of the model is: For a step change in feed concentration at time zero starting from steady state. Tank Example – Solution

Solve for CA(s)

If we like, we can rearrange to the form: This is the solution in the Laplace domain. To find the solution in the time domain, we must invert the Laplace transforms

CSTR Example Solution

### inverse Laplace transform

– Can be determined using a complex integral

### easiest approach is “ table lookup”

### Use Table 4-1, entry 5

### Maple is good at inverting Laplace transforms too

### The Impulse Function

### limit of the pulse function (with unit area) as the width goes to zero and height becomes infinite transform

### CSTR – Impulse Response physically – dump some pure A into reactor, all at once input function Transform

### time response

Interpretation of Impulse Response dump a bag of reactant into the reactor in a very very short time

we see an instantaneous jump to a new concentration due to the impulse input

concentration then decays back to the original steady-state concentration

Time-Shifted Functions – Representation of Delays Laplace transform for function with time delay

Just pre-multiply by an exponential.

How could we prove this?

– change of variables in integration in expression for Laplace Transform (see p. 103 of Marlin, p. 115 in first ed.)

Reactor Example with Time Delay

Suppose we add a long length of pipe to feed…

– assume plug flow

– It will take a time period, q minutes, before the change in

concentration reaches the tank, and begins to influence cA

– delay differential equation

» difficult to solve directly in time domain

» easy to solve with Laplace transforms

Tank Example with Time Delay – Solutionresponse to step input in cA0 time response

Final Value Theorem An easy way to find out what happens to the output variable if we wait a long time. We don’t have to invert the Laplace transform!

Why is it true?

– Consider the Laplace transform of a time derivative now let s approach zero

provided dy/dt isn’t infinite between t= 0 and t®¥ (i. e y(t) is STABLE) This will be true if Y(s) is continuous for s³0

Using the Final Value Theorem – Step Response Reactor example – final value after a step input

What can we do with Laplace Transforms so far.

Take Laplace transforms of linear ODEs (in deviation variables).

### Substitute Laplace transform expressions for different kinds of inputs we are interested in:

– Steps, pulses, impulses (even with dead time)

### Solve for the output variable in terms of s.

### Invert the Laplace transform using Table 4. 1 to get the solution in the time domain.

### Find the final steady state value of the output variable, for a particular input change, even without inverting the Laplace transform.

### Laplace transforms are mostly used by control engineers who want to determine and analyze transfer functions.

### compact way of expressing process dynamics

### relates input to output

### p(s), q(s) – polynomials in s

### – q(s) will also contain exponentials if time delay is present

### Once we know the transfer function of the process, we can use it to find out how the process responds to different types of input changes: