Applying the laws of sines and cosines

Science, Mathematics



Applying the Laws of Sines and Cosines Applying the Laws of Sines and Cosines It is necessary in trigonometry to solve the triangle or to find all its sides and angles. The solution can be found in the following cases:

Two angles and the side between these angles are known (ASA);

Two angles and the side opposite to one of the angles are known (AAS);

Two sides and the angle opposite to one of the sides are known (SSA or ambiguous case);

Two sides and the angle between them are known (SAS);

All sides of the triangle are known (SSS) (Aufmann and Nation, 2014). The law of sines or the law of cosines can be used for the mentioned cases. The law of sines is applied for the cases ASA, AAS and SSA. According to the law, the length on the side in the triangle is proportional to the opposite angle or (Aufmann and Nation, 2014):

For AAS and ASA problems the third angle can be found subtracting known angles from 180°. The unknown sides are found from the proportion. For instance, b = (a*sin B)/sin A = (c*sin B)/sin C. For SSA problem or ambiguous case no solution, one solution and two solutions can be found. If sinus for the opposite angle for the one of the known side is greater than 1, there is no solution for the triangle. If there are possible two angles for the calculated sinus (for instance B and 180° - B), and the sum of known and calculated angles is less than 180°, two triangles can be solved. For both solutions of the ambiguous case the third angle is found and the rest sides are calculated (Aufmann and Nation, 2014).

SSS and SAS cases are solved applying the law of cosines. According to the law of cosines, the square of any side of the triangle is equal to the sum of

squares of the rest sides subtracting double product of these sides and a cosine of the angle between these sides (Aufmann and Nation, 2014). For instance,

For SSS case, the cosine of each angle can be found from the equation, and the corresponding angles can be calculated. For the SAS case the unknown side is found from the equation initially. Then, the rest angles are found using the procedure for SSS case (Aufmann and Nation, 2014).

Thus, the general guidelines for any triangle are to define the problem or the case (SSS, SSA) encountered. For the ambiguous case, the number of possible triangles has to be found. Afterward, the solution for the triangle can be found using the procedures described above.

References

Aufmann, R. & Nation R. (2014). Algebra and Trigonometry, (8th ed.). Stamford, CT: Cengage Learning.