Linear algebra

Science, Mathematics

LINEAR ALGEBRA QUESTIONS Question 7 marks] é1 2 ê 5 b ù ú é1 2 0 3 ù ê ú

The reduced row echelon form of

A = ê4

a17

- 22ú is equal to R = ê001

- 2ú .

ëêrow3úû

êë000

0 úû

(a) What can you say about row 3 of A? Give an example of a possible third row for A.

This is a null row representation that the results become zero due to the figures of R which are equivalent to zero.

(b) Determine the values of a and b.

{1, 2, 5, 1} {b} = {1, 2, 0, 3}

{4, 1, 17, -22} {a} = {0, 0, 1, -2}

B= 1

A = 0

(c) Determine the solution of the homogeneous system of equations Rx = 0 in parametric vector form.

Rx = 0

Row 3 – {0, 0, 0)

(d) What is the dimension of the column space of A? Do the columns of A span R3 ?

The dimension of the column is equal to 1 and the columns of A do not span

R

Question2[6 marks]

Suppose an economy has four sectors, Agriculture (A), Energy (E), Manufacturing (M), and Transportation (T). Sector A sells 10% of its output to E and 25% to M and retains the rest. Sector E sells 30% of its output to A, 35% to M, and 25% to T and retains the rest. Sector M sells 30% of its output to A, 15% to E, and 40% to T and retains the rest. Sector T sells 20% of its output to A, 10% to E, and 30% to M and retains the rest.

(a) Construct a diagram that shows the exchange between the three sectors.

{55, 10, 35, 0} {A} = {100}

{30, 10, 35, 25} {E} = {100}

{30, 15, 15, 40} {M} = {100}

{20, 10, 30, 40} {T} = {100}

(b) Determine the exchange table for this economy, where the columns describe how the output of each sector is exchanged among the four sectors.

A

E

M

T

55

10

35

0

30

10

35

25

30

15

15

40

20

10

30

40

(c) Denote the prices of the total annual outputs of the sectors by pA, pE, pM and pT respectively. Determine the equations that need to hold for the equilibrium prices for the four sectors.

Pa = 55A+ 10E+35M

Pa= 11A+2E+7M……………………………………………………..(i)

Pe = 30A + 10E +35M +25T

Pe = 6A + 2E + 7M + 5T…………………………………………..(ii)

Pm = 30A + 15E +15M +40T

Pm = 6A + 3E + 3M + 8T…………………………………………..(iii)

Pt = 20A + 10E + 30M + 40T

Pt = 2A + E + 3M + 4T……………………………………………..(iv)

(d) Find the equilibrium prices, if they exist.

11A+2E+7M = 100

6A + 2E + 7M + 5T = 100

6A + 3E + 3M + 8T = 100

2A + E + 3M + 4T = 100

5A= 100……………………………………………………………(v)

4A + 2E + 4T = 100…………………………………………(vi)

A= 95/4

Equilibrium = 23. 75

Question3[6 marks]

Balance the following chemical equation using the vector equation approach.

PbN6 + CrMn2O8 → Pb3O4 + Cr2O3 + MnO2 + NO

Solution:

PbN6 + CrMn2O8 → Pb3O4 + Cr2O3 + MnO2 + NO

15 PbN6 + 8 CrMn2O8 = 5 Pb3O4 + 4 Cr2O3 + 16 MnO2 + 90 N0

1218 PbN6 + 2233 CrMn2O8 = 406 Pb3O4 + 11 Cr203 + 4466 MnO2 + 7308 NO

Question4[6 marks]

For what values of h is

v 3 in Span {v1, v2}? Justify your answer.

(a) For what values of h is {v1, v2, v3} linearly dependent? Justify your answer.

Question5 [10 marks]

Consider the linear transformation T (x1 , x2 , x3) = (2x1 - 2x2 - 4x3 , x1 + 2x2 + x3)

(a) Find the image of (3, - 2, 2)

under T.

x coordinate goes from -3 to 2 then it moves a total distance of +5 in the x direction.

If the y coordinate goes from -2 to 2 then A moves a total distance of +4 in the y direction.

So the translation is (+5,+4)

Adding the x coordinate of B to the x coordinate of translation: 3+5= 8 in x direction

Adding the y coordinate of B to the y coordinate of translation: -2+4= 2 in y direction

So under the same translation, B(8, 2)

Therefore the image

(b) Does the vector (5, 3) belong to the range of T?

Yes because the above image range translates to (8, 2), an area that defines and includes the image range (5, 3) too.

(c) Determine the matrix of the transformation.

| x_1 y_1 1| | d| | y_1|

| x_2 y_2 1| | e| = | y_2|

| x_3 y_3 1| | f| | y_3|

(2x1 - 2x2 - 4x3) = (x1)

(x1 + 2x2 + x3) = (x2)

{2, -6) +(-2, 4)

T(8, 2)

(d) Is the transformation T onto? Justify your answer

Yes the transformation lie just within the defined image line

(e) Is the transformation one-to one? Justify your answer

No just because the transformation is not invertible

Question6[5 marks]

a) For each of the following matrices explain why the matrix is not invertible.

é 200 ù

êú

é 34

ê

- 6 ù

ú

é 5- 2

ê

15 ù

ú

i) ê 0

00 ú

ii) ê 7

21 ú

iii) ê 1- 43 ú

êë 9

30 úû

êë - 6- 8

12 úû

êë 21

6 úû

From the three matrices above, the matrix is not invertible since the determinant of the matrix is zero. More so, on the second and third matrices, the determinants of these two matrices translate to a negative figure clearly showing that the matrices are not invertible. In addition, a matrix is not invertible if and only if there is a linear dependence between rows, i. e. that one row is a linear combination of the others and that is the case for the three matrices provided.

b) Suppose A is an n×n matrix with the property that the equation Ax = 0 has only the trivial solution. Without using the Invertible Matrix Theorem, explain directly why the equation Ax = b must have a solution for each b in R^n

An nxn matrix A is called nonsingular or invertible if there exists an nxn matrix B such that

where In is the identity matrix. The matrix B is called the inverse matrix of A.

The equation Ax = b must be a solution for each b in Rn since its determinant can be found using other prevailing methods.