Patterns within systems of linear equations

Science, Mathematics

Jasmine Chai Grade 10 196298501 Patterns within systems of linear equations Systems of linear equations are a collection of linear equations that are related by having one solution, no solution or many solutions. A solution is the point of intersection between the two or more lines that are described by the linear equation. Consider the following equations: x + 2y = 3 and 2x – y = -4. These equations are an example of a 2x2 system due to the two unknown variables (x and y) it has. In one of the patterns, by multiplying the coefficient of the y variable by 2 then subtract the coefficient of x from it you will be given the constant.

As a word equation it can be written like so with the coefficient of x as A and coefficient of y as B and the constant as C, 2B – Ax = C. This can be applied to the first equation (x + 2y = 3) as 2(2) – 1 = 3. To the second equation (2x – y = -4), it is -1(2) – 2 = -4. By using matrices or graphs, we can solve this system. Regarding other systems that also has such as pattern, it should also have the same solution as the two examples displayed. For instance, 3x + 4y = 5 and x -2y = -5, another system, also displays the same pattern as the first set and has a solution of (-1, 2).

Essentially, this pattern is indicating an arithmetic progression sequence. Arithmetic progression is described as common difference between sequences of numbers. In a specific sequence, each number accordingly is labelled as an. the subscript n is referring to the term number, for instance the 3rd term is known as a3. The formula, an = a1 + (n – 1) d, can be used to find an, the unknown number in the sequence. The variable d represents the common difference between the numbers in the sequence. In the first equation (x + 2y = 3) given, the common differences between the constants c – B and B – A is 1.

Variable A is the coefficient of x and variable b represents the coefficient of y, lastly, c represents the constant. The common difference of the second equation (2x – y = -4) is -3 because each number is decreasing by 3. In order to solve for the values x and y, you could isolate a certain variable in one of the equations and substitute it into the other equation. x + 2y = 3 2x – y = -4 x + 2y = 3 * x = 3 – 2y * 2(3 – 2y) – y = -4 * 6 – 4y – y = -4 * 6 – 5y = -4 * -5y = -10 * y = 2 Now that the value of y is found, you can substitute 2 in as y in any of the equations to solve for x. x + 2y = 3 x + 2(2) = 3 * x + 4 = 3 * x = 3 – 4 * x = -1 Solution: (-1, 2) Even though the solution has already been found, there are many different ways to solve it, such as graphically solving it. By graphing the two linear lines, you can interpolate or extrapolate if necessary to find the point where the two lines intersect. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Graph 1 Graph 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Just from the equations given, it is not in a format where it can be easily graphed. By changing it into y= mx + b form, the first equation will result as y = - (1/2) x + 3/2 or y = -0. 5x + 1. 5 and the second equation will result as y = 2x + 4. The significance of the solution is that it is equal to the point of intersection as shown on Graph 1. This can then allow the conclusion that the solution of the two linear equations is also the point of intersection when graphed. According to this arithmetic progression sequence, it could be applied to other similar systems.

For instance, the examples below demonstrates how alike 2x2 systems to the previous one will display a similarity. Example 1: In the first equation the common difference between (3, 4 and 5) is 1. In the second equation, the common difference is -3. The common differences in these equations are exact to the previous example. 3x + 4y = 5 x – 2y = -5 x – 2y = -5 * x = 2y – 5 (Substitution) 3x + 4y = 5 * 3(2y – 5) + 4y = 5 * 6y – 15 + 4y = 5 * 10y – 15 = 5 * 10y = 20 * y = 2 (Substituting y) x – 2y = -5 * x – 2(2) = -5 * x – 4 = -5 * x = -5 +4 * x = -1 Solution: (-1, 2)

Example 2: In the first equation below, it has a common difference of 18 for (2, 20 and 38). For the second equation, in (15, -5 and -25), it has a common difference of -20. In this example, the system is solved graphically. 2x + 20y = 38 15x – 5 y = -25 Solution: (-1, 2) | Graph 2 Graph 2 | | |

From the examples given above that are very similar to the first system, we can conclude that there is something common between them, that is the point of intersection or the values of x and y. That would imply that the x and y values and the point of intersection will always be (-1, 2) for all systems that follow arithmetic progression sequences. Due to that similarity, an equation that can be applied to these types of equations can be made. If the first coefficient of the first equation is identified as A and the common difference is c, an equation such as, Ax + (A + c) y = A + 2c, is made.

This equation is so, because it is describes an arithmetic sequence, where the coefficients and constant are increasing by one in response to the coefficient before. In the second equation of the system, another equation can be made relatively the same to the first, with exceptions of different variables used. If B is used to represent the first coefficient of the second equation and d is used as the common difference, the equation, Bx + (B + d) y = B + 2d is created. With 2 equations, we have now created a system; to solve the system we can use the elimination method.

This method is used to eliminate certain variables in order to find the value of another variable. After doing so, you could substitute in the value for the found variable and solve for the other(s). Ax + (A + c) y = A + 2c Bx + (B + d) y = B + 2d In order to use the elimination method, you must make the coefficient of x or y the same depending on which one you would like to eliminate. In this case, we will start by eliminating x. To proceed to do so, we must first multiply the first equation by B and the second equation by A: ABx + (AB + Bc) y = AB + 2Bc ABx + (AB + Bd) y = AB + 2Bd

After we have made the coefficient of x the same for both equations, we can now subtract the equations from one another: ABx + ABy + Bcy = AB + 2Bc ABx + ABy + Bdy = AB + 2Bd * Bcy - Bdy = 2Bc – 2Bd To find the value of y, we must isolate the variable y. Bcy - Bdy = 2Bc – 2Bd * y(Bc – Bd) = 2(Bc – Bd) * y = 2 Now that the value of y is found, to find the value of x is to substitute the value of y, which is 2, into any equation that includes that variable x and y. Bx + (B + d) y = B + 2d * Bx + (B + d) 2 = B + 2d * Bx + 2B + 2d = B + 2d * Bx + 2B – B = 2d – 2d * Bx + B = 0 * Bx = -B * x = -1

To conclude the results of the equations above, it is making thee statement that all 2x2 systems that display an arithmetic progression sequence, which has a common difference between the coefficients and constant, it will have a result, point of intersection, of (-1, 2). To confirm that this is correct, the example systems below will demonstrate this property: Equation 1 (common difference of 8): 2x + 10y = 18 Equation 2 (common difference of 3): x + 4y = 7 Substitution Method x + 4y = 7 * x = 7 – 4y Substitute 2x + 10y = 18 * 2 (7 – 4y) + 10y = 18 * 14 – 8y +10y = 18 * 14 + 2y = 18 2y = 18 – 14 * 2y = 4 * y = 2 Substitute x + 4y = 7 * x + 4(2) = 7 * x + 8 = 7 * x = 7 – 8 * x = -1 Solution: (-1, 2) Once again from the example above, it displays that the solution or the point of intersection is identified as (-1, 2). From previous examples, all have a common difference that is different from the other equation involved in that system. In the following example, it will experiment whether having the same common difference will make a difference in the result. Equation 1 (common difference of 3): 2x + 5y = 8 Equation 2 (common difference of 3): x + 3y = 6 Graph 3 Graph 3

As you can see on the graph, it shows that the two lines do not intersect at (-1, 2) even though it is a 2x2 system that has a common difference in both equations, meaning that the intersection at (-1, 2) can only be applied to systems that has 2 different common differences. To conclude, all 2x2 systems that follow arithmetic progression sequence with different common difference have a solution of (-1, 2). Furthermore, now that it is known that there is a certain pattern for a specific type of system, if this property is applied to a 3x3 system, with 3 different variables can it still work?

Consider the following 3x3 system, (x + 2y + 3z = 4), (5x + 7y + 9z = 11) and (2x + 5y + 8z = 11). In this system, it has similar patterns to the 2x2 systems above due to its arithmetic progression. In the first equation, it has a common difference of 1 and the second equation has a common difference of 2 and lastly, the third equation has a common difference of 3. To solve this system, we can solve it using the method of elimination or matrices. Equation 1 (common difference: 1): x + 2y + 3z = 4 Equation 2 (common difference: 2): 5x + 7y + 9z = 11

Equation 3 (common difference: 3): 2x + 5y + 8z = 11 Elimination Method To eliminate the variable x, we must first start by making the coefficients of x in two equations the same. We can do so by finding the lowest common multiple of the two coefficients and multiplying the whole equation by it. Equation 1: x + 2y + 3z = 4 * 2(x + 2y + 3z = 4) * 2x + 4y + 6z = 8 We can eliminate the variable x now that the coefficients of x in both equations are the same. To eliminate x, we can subtract equation 3 from equation 1. Equation 1 and 3: 2x + 4y + 6z = 8 2x + 5y + 8z = 11 -y -2z = -3 After eliminating x from two equations to form another equation that does not involve x (-y -2z = -3), another equation that does not involve x must be made to further eliminate another variable such as y or z. Equation 1: x + 2y + 3z = 4 * 5(x + 2y + 3z = 4) * 5x + 10y + 15z = 20 We can eliminate the variable x now that the coefficients of x in both equations are the same. To eliminate x, we can subtract equation 2 from equation 1. Equation 1 and 2: 5x + 10y + 15z = 20 - 5x + 7y + 9z = 11 3y + 6z = 9

Now that two different equations that do not involve x ((-y -2z = -3) and (3y + 6z = 9)) are created, we can find the common coefficient of y and eliminate it to find the value of the variable z. Let (-y -2z = -3) to be known as equation A and (3y + 6z = 9) will be known as equation B. Equation A: -y -2z = -3 * 3(-y -2z = -3) * -3y -6z = -9 Equation A and B: -3y -6z = -9 + 3y + 6z = 9 0 = 0 As you can see from the result, 0 = 0, this is indicating that the system either has many solutions, meaning a collinear line or no solution, where all the lines do not intersect together at a specific point.

Even if you attempt to isolate a different variable it will still have the same result. For instance, using the same equations above, you eliminate the variable y first as displayed below. Equation 1 (common difference: 1): x + 2y + 3z = 4 Equation 2 (common difference: 2): 5x + 7y + 9z = 11 Equation 3 (common difference: 3): 2x + 5y + 8z = 11 Elimination Method Equation 1: x + 2y + 3z = 4 * 7(x + 2y + 3z = 4) * 7x +14y + 21z = 28 Equation 2: 5x + 7y + 9z = 11 * 2(5x + 7y + 9z = 11) * 10x + 14y + 18z = 22 Equation 1 and 2: 7x +14y + 21z = 28 - 10x + 14y + 18z = 22 3x + 3z = 6 Equation 1: x + 2y + 3z = 4 * 5(x + 2y + 3z = 4) * 5x +10y + 15z = 20 Equation 3: 2x + 5y + 8z = 11 * 2(2x + 5y + 8z = 11) * 4x + 10y +16z = 22 Equation 1 and 3: 5x +10y + 15z = 20 - 4x + 10y +16z = 22 x - z = -2 Two equations have been made that has already eliminated the variable y. Let (-3x + 3z = 6) be equation A and let (x - z = -2) be equation B. Doing this, is in attempt to solve for variable x. Equation A: -3x + 3z = 6 Equation B: x - z = -2 * 3(x - z = -2) * 3x - 3z = -6 Equation A and B: -3x + 3z = 6 + 3x - 3z = -6 0 = 0

As you can see the result, it is the same even if you try to solve another variable, from that we can confirm that this system has either no solution or infinite solutions, meaning that they are collinear lines. Furthermore, because this is a 3x3 system, meaning that it has three different variables, such as x, y and z, graphing it will also be very different from a graph of a 2x2 system. In a 3x3 system, the graph would be a surface chart, where the variable z allows the graph to become 3D. From this, we can conclude 3x3 systems that follow an arithmetic progression will always have either no solution or infinite solutions.

This is saying that all linear equations do not intersect together in one point or they do not intersect. A way to prove this is through finding the determinant. The determinant is a single number that describes the solvability of the system. To find the determinant of all 3x3 systems that possesses arithmetic progression, we can start by creating a formula. Allow the first coefficient of the first equation be A and the second equation's first coefficient be B and lastly, the first coefficient of the third equation be C.

The common difference of equation one will be c, the common difference of equation two will be d, and the common difference of equation e will be e. This can be described through the following equations: 1. Ax + (A + c) y + (A + 2c) z = (A + 3c) 2. Bx + (B + d) y + (B + 2d) z = (B + 3d) 3. Cx + (C + e) y + (C + 2e) z = (C + 3e) When developing a matrix to find the determinant, you must have a square matrix. In this case, we do not have a square matrix. A square matrix is where the number of rows and columns are equal, for example, it could be a 2x2, 3x3, or 4x4. Looking at the equations, it is a 3x4 matrix; as a result it must be rearranged.

Below is the rearranged matrix of the equations above. x A (A + c) (A + 2c) (A + 3c) y B (B + d) (B + 2d) = (B + 3d) z C (C + e) (C + 2e) (C + 3e) To find the determinant, you must find 4 values from the 3x3 matrix that helps find the determinant of A, B and C. In this case, if you were to find the values for A, you would cover the values that are in the same row and column as A, like so, A (A + c) (A + 2c) B (B + d) (B + 2d)

C (C + e) (C + 2e) You would be left with four separate values that can be labelled as A, B, C and D. Respectively to the model below: a b c d In order to find the determinant you must find the four values for A, (A + c) and (A +2c). To find the determinant the equation ad – cb is used. The equation in this situation would be like the one below: A[(B + d)(C + 2e) – (C + e)(B + 2d)] – (A + c)[B(C + 2e) – C(B + 2d)] + (A +2c)[B(C + 2e) – C(B + 2d)] Expand * = A(BC – BC + Cd – 2Cd + 2Be – Be + 2de – 2de) – (A + c)(BC – BC + 2Be – 2Cd) + (A + 2c)(BC – BC + 2Be – 2Cd) Simplify 2ABe – 2ABe + 2ACd – 2ACd + 2Ccd – 2Ccd + 2Bce – 2Bce * = 2ABe – 2ABe + 2ACd – 2ACd + 2Ccd – 2Ccd + 2Bce – 2Bce * = 0 As it is visible, above it shows that the determinant found in this type of matrix is zero. If it is zero, it means that there are infinite answers or no answer at all. Usingtechnology, a graphing calculator, once entering a 3x3 matrix that exhibits arithmetic progression, it states that it is an error and states that it is a singular matrix. This may mean that there is no solution. To conclude, there is no solution or infinite solution to 3x3 systems that exhibit the pattern of arithmetic sequencing.

This can be proved when the sample 3x3 system is graphed and results as a 3D collinear segment. As well as the results from above when a determinant is found to be zero proves that 3x3 systems that pertains an arithmetic sequence. Arithmetic sequences within systems of linear equations are one pattern of systems. Regarding other patterns, it is questionable if geometric sequences can be applied to systems of linear equations. Consider the following equations, x + 2y = 4 and 5x – y = 1/5. It is clear that the coefficients and constants have a certain relation through multiplication.

In the first equation (x + 2y = 4), it has the relation where it has a common ratio of 2 between numbers 1, 2 and 4. For the second equation (5x – y = 1/5), it has a common ratio of -1/5 between 5, -1 and 1/5. The common ratio is determined through the multiplicative succession from the previous number in the order of the numbers. When the equations are rearranged into the form y= mx+b, as y = - ? x + 2 and y = 5x – 1/5, there is a visible pattern. Between the two equations they both possess the pattern of the constant, where constant a is the negative inverse of constant b and vice versa.

This would infer that if they are multiplied together, as follows (-1/2 x 2 = -1 and 5 x -1/5 = -1), it will result as -1. With equations that are also similar to these, such as the following, y = 2x – 1/2, y = -2x + 1/2, y = 1/5x – 5 or y = -1/5x +5. Displayed below, is a linear graph that shows linear equations that are very similar to the ones above. Graph 4 Graph 4 From the graph above, you can see that the equations that are the same with exceptions of negatives and positives, they reflect over the axis and displays the same slope.

For instance, the linear equations y = 2x -1/2 and y=-2x +1/2 are essentially the same but reflected as it shows in the graph below. Also, all equations have geometric sequencing, which means that they are multiplied by a common ratio. Secondly, the points of intersection between similar lines are always on the x-axis. Graph 5 Graph 5 Point of intersection: (0. 25, 0) Point of intersection: (0. 25, 0) To solve a general 2x2 system that incorporates this pattern, a formula must be developed. In order to do so, something that should be kept in mind is that it must contain geometric sequencing in regards to the coefficients and constants.

An equation such as, Ax + (Ar) y = Ar2 with A representing the coefficients and r representing the common ratio. The second equation of the system could be as follows, Bx + (Bs) y = Bs2 with B as the coefficient and s as the common ratio. As a general formula of these systems, they can be simplified through the method of elimination to find the values of x and y. Ax + (Ar) y = Ar2 Bx + (Bs) y = Bs2 Elimination Method B (Ax + (Ar) y = Ar2) * BAx + BAry = BAr2 A (Bx + (Bs) y = Bs2) * ABx + ABsy = ABs2 Eliminate BAx + BAry = BAr2 - ABx + ABsy = ABs2 BAry – ABsy = BAr2 – ABs2 ABy (r – s) = AB (r2 - s2) * y = (r + s) Finding value of x by inputting y into an equation ABx + ABsy = ABs2 * ABx + ABs(r + s) = ABs2 * ABx = ABs2 – ABs(r +s) * x = s2 – s(r +s) * x = s2 – s2 – rs * x = rs To confirm that the formula is correct, we can apply the equation into the formula and solve for x and y and compare it to the results of graph 4. The equations that we will be comparing will be y = 5x – 1/5 and y = -1/5x + 5. The point of intersection, (1, 4. 8) of these equations is shown graphically on graph 4 and 6. The common ratio (r) of the first equation is -0. and the common ratio, also known as s in the equation of the second equation is 5. X = - (-0. 2 x 5) = 1 Y = (-0. 2 + 5) = 4. 8 As you can see, above, the equations are correctly matching the point of intersection as shown on the graphs. Due to such as result, it is known that it can now be applied to any equations that display geometric sequencing. Graph 6 Graph 6 Resources: 1. Wolfram MathWorld. Singular Matrix. Retrieved N/A, from http://mathworld. wolfram. com/SingularMatrix. html 2.

Math Words. Noninvertible Matrix. Retrieved March 24, 2011 from, http://www. mathwords. com/s/singular_matrix. htm