

Graphing

Science, Mathematics



Linear equations Given equations Equation M is $y = x + 4$ and Equation of line Z is $y = 2x$ Parallel line to line M passes through coordinates $(-7, 1)$ and perpendicular line to line Z pass through coordinates $(4, 2)$.

Finding the equation of the parallel line to line M

In the equation of the origin line M $y = x + 4$, the gradient is 1. Parallel lines have same gradients. Therefore, the gradient of the parallel line is 1. The gradient is calculated by finding the change in Y-axis divided by change in X-axis. Assume the parallel line pass through another coordinates (x, y) .

Gradient of parallel line = change in y-axis/change in x-axis

$$\text{Gradient} = \frac{y-1}{x+7} = 1$$

$y - 1 = 1(x + 7)$ solve the equation by making y the subject of the formula.

$$y = x + 8$$

The equation of the parallel line to the given line M is $y = x + 8$

Finding the equation of the perpendicular line to given line Z.

Line Z is given as $y = -\frac{1}{2}x + 1$ and the perpendicular line to the line Z passes through coordinates $(4, 2)$. The Gradient of line Z is $-\frac{1}{2}$ and y- intercept is 1. The product of gradients of perpendicular lines is -1. Therefore, $(-\frac{1}{2}) \times (G_2) = -1$. G_2 is the gradient of the perpendicular line.

$$G_2 = -1 \div (-\frac{1}{2})$$

$$G_2 = 2. \text{ (Gradient of the perpendicular line)}$$

Assume that the perpendicular line passes through point (x, y) and $(4, 2)$

Gradient = change in y-axis/ change in x-axis

$$2 = \frac{y-2}{x-4}$$

$y - 2 = 2(x - 4)$. Make y the subject of the formula

$$Y = 2x - 6.$$

The equation of the perpendicular line to line Z is $Y = 2x - 6$

Discuss the steps necessary to carry out each activity. Step of finding the equation of the line.

Assume the line passes through points A (4, 6) and N (2, 3)

• Find the gradient.

$$Y = mx + c$$

$$\text{Gradient (m)} = \frac{6-3}{4-2} = \frac{3}{2}$$

$$Y = \frac{3}{2}x + c$$

Find y-intercept

Take the coordinates of one point. E. g. A (4, 6)

$$6 = \frac{3}{2}(4) + c$$

$$6 = 6 + c$$

$$0 = c$$

Write the equation of the original line.

$$Y = \frac{3}{2}x + 0$$

Equation of a parallel line

An equation of the parallel line to the original line is determined through the following.

Original equation is equal to $Y = \frac{3}{2}x + c$ and the gradient of original is $\frac{3}{2}$.

The gradient of parallel is also $\frac{3}{2}$. Therefore, the equation of a parallel line to the original is $y = \frac{3}{2}x + c$.

Equation of a perpendicular line

An equation of the perpendicular line to the original line is determined through the following.

Original equation is equal to $y = \frac{3}{2}x + c$ and the gradient is $\frac{3}{2}$. The product

of gradient of perpendicular and original line is -1 . Therefore, $3/2 (m_2) = -1$.

m_2 becomes $-2/3$. The equation of perpendicular is therefore $y = -2/3 x + c$.

Describe briefly what each line looks like in relation to the original given line.

Parallel line has the same gradient as the original given line hence they are similar.

Perpendicular line meets the original line at a right angle (90°)

What does it mean for one line to be parallel to another?

One line is parallel to another if the lines can never meet when extended in both directions. Parallel lines have the same gradient or slope hence they never meet each other. Parallel lines are easily determined by comparing gradients or slopes of each given lines in their equations. The equations are normally in the form of $y = mx + c$, whereby y represents the values of y -axis in the line and x represents the values of x -axis. m represents the gradient or the slope of the line while c represents the y -intercept. Y -intercept is a point where the line cuts the y -axis. X -intercept is a point where the line cuts the x -axis.

What does it mean for one line to be perpendicular to another?

Lines are said to be perpendicular to each other if the product of their gradients is equal to -1 . This means that perpendicular lines meet at a right angle, that is, the angle between perpendicular lines is 90° . The ordered pair or the coordinate of their meeting point is equal. Given the equation of a line, one can determine the gradient of the perpendicular line by finding the negative reciprocal of the gradient of the original line. Coordinate $(0, 0)$ is referred to as the origin. It is the meeting point of y -axis and x -axis.

Coordinates or ordered pair must be enclosed in a bracket.

Reference

Kelley. C. T (1995). Linear equations. Iterative methods for linear and nonlinear equations. Philadelphia. Society for Industrial and Applied Mathematics.