The function to be integrated essay

Q1. The function to be integrated numerically isa) Graph of the function in the region of integration is given below.

This graph has been generated using MS Office Excel by calculating the value of the function for values of x in the range 0 to 1 at the step size of 0. 1. b)c) Using Mid-Point RuleLet us take step size h = 0.

5Now the value of the integral will beM2 = h*[f(0. 25) + f(0. 75)] = 0. 5* (0. 964929 + 0. 836029) = 0. 909169Similarly, for step size h = 0.

25M4 = 0. 25*[f(0. 125) + f(0. 375) + f(0. 6250) + f(0.

875)] = 0. 904098Similarly, for step size h = 0. 125M8 = 0. 125 * [f(0. 0625) + f(0. 1875) + f(0.

3125) + f(0. 4375) + f(0. 5625) + f(0.

6875) + f(0. 8125) + f(0. 9375)] = 0.

902808Using Trapezium rule withStep size h = 0. 5The integral will beT2 = (h/2)*[f(0) + 2*f(0. 5) + f(1. 0)] = 0. 25 * 3. 554699 = 0. 888675Step size h = 0.

25The integral will beT4 = (h/2)*[f(0) + 2*[f(0. 25) + f(0. 5) + f(0. 75)] + f(1. 0)] = 0.

125 * 7. 191373 = 0. 898922Step size h = 0.

125The integral will beT8 = (0. 125/2) * 14. 424154 = 0. 901510Simpson’s Method: Simpson’s method givesWhere n is the number of steps, M and T represent value of the integral corresponding to the n number of steps. In this case I have chosen n to be 10; therefore, c) Extrapolation to infinity is a geometric series with a = (first term) and r = ¼Therefore, d) The value of integral has been calculated up to six decimal places to minimize truncation error arising out of large number of steps. Q2. (i) The finite difference table was constructed using MS Office Excel.

It is presented below: xifiDfiD2fiD4fiD6fi-20. 0142640. 331327-10. 3455910. 3230830. 654409-1.

63190101. 000000-1. 3088183.

263802-0. 6544091. 63190110. 3455910.

323083-0. 33132720. 014264Because h = 1 here, therefore, the Newton Interpolating polynomial will be(ii) Therefore,(iii) Using Midpoint rule Let us take a step size h = 0. 20; thenM4 = 0. 20*[f(0. 10) + f(0.

30) + f(0. 50) + f(0. 70) + f(0. 90)] = 0. 20 * 3. 682 = 0.

74Using Trapezium rule and again taking step size h = 0. 20T5 = (h/2)*[f(0)+2*[f(0. 20) + f(0. 40) + f(0.

60) + f(0. 80)] + f(1. 0)]   = (0. 20/2) * 7.

327243 = 0. 73Now, using Simpson’s ruleThus, it can be seen that N(x) is a good approximation of f(x) because the value of the integral for the function as well as the corresponding N(x) is agreeing with each other with great accuracy. Q3. Given equation isTo solve this equation in 0 to 5 range, what is required is to compute roots of f(x) in the same range such that f(x)

= f’(x) = Value of f(x) was computed in (0, 5) range at step size of 0.

1 and it was found that value of f(x) changes its sign from +ve to –ve between x = 1. 0 and x = 1. 2 and again from –ve to +ve between x = 2.

8 to x = 2. 9 and nowhere else. Thus it can be concluded that the given equation has two solutions in (0, 5) range – one solution is near x = 1 and another near x = 2.

8. Further iterations will be made around these two points to get more accurate solution points. Iteration between x = 1. 0 and x = 1.

1x = 1 f(x) = 0. 121029; x = 1. 1 f(x) = -0. 01962Therefore, x0 = 1. 0 f(x) = -0.

01535 x1 = 1. 05 f(x) = 0. 047458 x2 = 1. 075 f(x) = 0. 013142 x3 = 1. 0875 f(x) = -0.

00343 x4 = 1. 08125 f(x) = 0. 004809 x5 = 1. 084375 f(x) = 0. 000679Thus solution of the given equation up to six decimal places is x = 1. 084375Iteration between x = 2. 8 and x = 2.

9x = 2. 8 f(x) = -0. 01535; x = 2. 9 f(x) = 0. 068078Therefore, x0 = 2. 8 f(x) = -0.

01535 x1 = 2. 85 f(x) = 0. 025899

x2= 2. 825 f(x) = 0. 005152 x3 = 2. 8125

f(x) = -0. 00513 x4 = 2.

81875 f(x) = 0. 00000355 x5 = 2. 815625 f(x) = -0. 00256Thus solution of the given equation up to six decimal places is x = 2. 815625. Hence the solutions of the given equation in the (0, 5) range are x = 1.

084375 and 2. 815625. Q4. (i) Givenxx0 = 0x1 = 0. 5×2 = 1. 0f(x)10.

928533510. 69763147Therefore, This leads toM1 = 1 * f(0. 5) = 0. 9285T1 = (1/2)[f(0) + f(1)] = 0. 8488Hence, Comparing the answers it appears that S1 for(iii) Based on these observations it can be inferred that Sn for