# Period of oscillation of a simple pendulum 

## ASSIGN BUSTER

To find out what factors affect the period of oscillation of a simple pendulum. I hope to find what these factors are by varying factors such as angle of release, mass of pendulum bob and length of pendulum.

Background Information

## * Pendulum

This is a device which consists of an object (pendulum bob) suspended from a fixed point that swings back and forth whilst it is under the influence of gravity. Due to the constant at which pendulums swing, they are excellent time measuring pieces and eventually led to the invention of the pendulum clock, which was the most accurate time measuring piece at that time.

## * Galileo’s Formula

The acceleration due to gravity of an object in freefall was a very important notion for Galileo to realise. There is a simple law that states the distance travelled in freefall is proportional to the square of the time elapsed.

It was during a Sunday mass in Pisa when he noticed a chandelier above him swinging to and fro. Using his pulse as a clock, Galileo saw that the period of the swing was independent of how far it swung. Only the length of the pendulum made any difference to the time required for a swing.

This formula allows the period of a pendulum to be calculated:
$P=2 ? ?(1 / g)$

* P is the period of oscillation of the pendulum (in seconds).
* $L$ is the length of the pendulum (in metres).
* G is the acceleration of gravity (on Earth this is counted as 9.8).
* ? has the value of $3.14159 .$. etc.

Prediction and Hypothesis

Length is a certain factor, which will determine the time of oscillation of a pendulum. It is difficult to state why this is, but it helps to actually visualise a pendulum swinging. The shorter the pendulum is, the faster it will oscillate and visa versa - the longer the pendulum is, the longer it will take for each oscillation to take place. This is because of the effect gravity has on the pendulum. In a short pendulum, it must swing quickly because it would defy gravity otherwise. As it is difficult to explain this concept, I shall draw a diagram, which may help to explain the situation.

As you can see, pendulum ' $b$ ' has a further distance to travel than pendulum ' $a$ '. Mathematically, it is possible to work out the distances using the following formula:
? d (diameter) ? (360ї $i^{1 ⁄ 2}$ ?? xï $i^{1 ⁄ 2}$ )

If this formula is applied to both pendulums, distance ' a' equates to 1.57 cm and distance ' b ' equates to 3.14 cm (double the distance of distance " $a^{\prime}$ ). If mass does not affect gravitational acceleration (which I will discuss later on in the hypothesis), then the pendulums will swing at the same speed. Therefore, the pendulum should take twice as long to complete one
oscillation where the length is double. Consequently, time of oscillation is certainly affected by the length of the pendulum.

Energy is converted into heat in order to overcome air resistance. Eventually, the pendulum will come to a halt. In theory, a constant swinging motion could only be achieved inside a vacuum, and one major source of gravity. If no air resistance can act on the pendulum, then none of the energy from the swinging motion is converted into heat due to friction as the pendulum moves. Therefore it is air resistance and pivot resistance that pendulums lose their energy from leading to the loss of height gained each time an oscillation is completed. However, pendulums have to overcome the air resistance, so my experiments will not be completely accurate.

The mass of an object should not affect the rate of oscillation if the air resistance is equal. For example, two cannon balls that are dropped from the same height, but both have different masses, in theory; they should hit the ground at exactly the same time. It is only air resistance, which prevents objects from falling at a uniform speed. Another example of this would be a pebble and a feather at the same mass. Obviously, the feather would be considerably slowed if there were air resistance acting on it. However in a vacuum, both the pebble and the feather would fall at exactly the same speed, as would any other object. As the rate of oscillation is linked to gravity, if I had two pendulum bobs, one made out of polystyrene and one out of lead, both of identical in size and shape, there would be absolutely no difference in the rate of oscillation. Therefore, I predict that both density and mass will have absolutely no effect on the oscillation of a pendulum.

Below is a diagram showing the forces involved when a simple pendulum swings.

In theory, both pressure and temperature will have a minute effect on the rate of oscillation. This is because the higher the pressure, the more air molecules there are to act on the pendulum. The bob will need to exert a greater level of force to overcome the resistance of the air. Temperature will also affect the density of the air particles. The hotter it is, the less dense the air will be, and therefore meaning there will be fewer air particles to overcome. The hotter the temperature it is, the more efficient the pendulum is. However, both of these variables are very small and will not a very big effect on the experiment.

It is difficult to say whether angle of release will have any effect on the rate of oscillation or not. If the pendulum bob is released at an angle of $45 \mathrm{i}^{1}{ }^{1 / 2}$ instead of $25 i{ }^{\circ}{ }^{112} 2$, then there will be a difference in speed. If the length is kept the same, then in theory, each oscillation should be identical in time. But due to gravitational acceleration, the larger the angle, the faster the pendulum will go. As speed increases, so does the amount of air resistance. For example if one swims, it is difficult to go very fast as there is increased resistance upon that person as they go faster, so much more energy has to be exerted to travel at the higher speed. The slower an object goes, the less air resistance it experiences, so the energy can be exerted for a longer time. If the pendulum is released from a higher height, and then it will be decelerated at an increased rate, consequently the time of each oscillation should be exactly the same, whether the pendulum is released from 45ï̀ ${ }^{1 ⁄ 2}$ or $25 i{ }^{2}{ }^{1 / 2}$.

Below is a diagram, which shows how potential energy (p. e) and kinetic energy (k. e) are continually interchanged in a pendulum. The diagram shows how at the point of release, the energy of the bob is all p. e. Then as it passes through the central position, it is all k. e. As the end of the swing is reached, the enrgy is all p. e. Between these positions, there is usually a mixture of p. e. And k. e. Eventually, the energy is converted into heat as the pendulum overcomes air resistance.

Plan and Apparatus

Apparatus to be used

* String
* Pendulum bob
* Boss and Clamp
* Stopwatch
* Ruler
* Protractor
* Masses
* Cork


## Apparatus Diagram

Factors of Oscillation That I will investigate: -

* Length of pendulum
* Mass of pendulum bob
* Angle of release (amplitude)

Method for Each Factor of the Investigation: -

* Length of Pendulum

This will be investigated by varying the length of the string. By timing how long it takes for a complete oscillation to take place (an oscillation is from the starting point back to the starting point again. I will count the time for ten oscillations and then divide the resulting figure by ten to get an average time. The pendulum will be released from $45 i{ }^{\circ}{ }^{11 / 2}$ each time so that this will be a fair test. I will repeat the experiment at 5 centimetre intervals. Starting at 5 cm through to 75 cm ( 15 results will be collected). I shall be able to analyse the results to see if any patterns emerge and draw scatter-graphs to show the results. I shall also compare the results to Galileo's theoretical answer to see how close my answer is to the theoretical answer. I will then be able to state how reliable my method of testing is and how it could be improved.

* Mass of Pendulum Bob

This will simply be investigated by placing masses onto the string via a hook and timing ten oscillations as before and then dividing this result by ten. 50 gramme masses will be used, with a maximum mass of 350 g . There will be a total of seven results. To make the test a fair one, each experiment will have
a length of 50 cm and an angle of release of $30 \ddot{i}^{1 / 2}$. This should ensure that the results are accurate enough to find out if the mass of the pendulum bob affects the oscillation. As stated before, I shall draw scatter-graphs etc and draw up tables to see if any patterns emerge.

## * Angle of Release

This experiment will be done in a similar style to the others. Ten oscillations will be timed and as before the answer will be worked out by dividing by ten. To keep this experiment fair, I shall keep the length at 40 cm and the pendulum bob used will be identical each time. The experiment will be repeated at $5 i{ }_{i} \dot{1}^{1 / 2}$ intervals, ranging from $5 i{ }_{i} i^{1 / 2}$ to $100 i ̈ i^{11 / 2}$. As mentioned before, I shall analyse the results and see if any patterns emerge. Graphs will be drawn to show any patterns that emerge.

There are no safety precautions that need to be taken into account in this experiment, only common sense should be observed.

Results

In the results section, I shall present information in the form of tables and graphs (e. g. scatter-graphs, bar charts etc). Hopefully, I shall be able to explain or give reasons for the results that I achieve and why the pendulum acts in this way. I will also observe any patterns and explain why they are occuring. I shall also make comparisons to the theoretical answers put forward by Galileo

[^0]Here, the results are very good, with no anomalous, unexpected results. The results that I had were very accurate except for the lengths of 20 cm and 65 cm, which both suffered the worse scores compared to Galileo's theoretical answer. They scored a difference of -0.027 and -0.039 respectively. However, the results were pretty accurate, considering the results were only a few split seconds off being absolutely correct. In fact one result, 40 cm had a result where the theoretical answer was achieved. By totalling together the differences between the theoretical answer and my answer, I can see how incorrect the results are as an average. On average, the results are out by 0.0675 seconds. This is only a small amount of time, however, if I had my answers exactly the same as the theoretical answers, then the total would equal zero.

I have drawn a scatter-graph to show these results. The scatter-graph shows the results of my experiment (in red) and the theoretical results (in green). The green dots at which the red dot cannot be seen, have the best results as this shows that the results are close to the theoretical answer. Those that are not so close to the corresponding dot, are not such accurate results.

Following this graph is a chart showing the results of the following expression: -

L ? tï $¿^{1 ⁄ 2} 2$ (Length is inversely proportional to time squared). This can be rearranged to form a constant figure which will apply to a certain length and time. It is rearranged to form $L=k t i i^{1 / 2}$.

This can be rearranged so that ' $k$ ' can be worked out by ' dividing length by time squared'. The chart gives this as a constant figure that applies to very length or time.

Scatter-graph that shows results of the experiment that investigates length.

Chart that shows how length is directly proportional to time squared.

The results of this chart can be interprted in various ways. An average for ' $k$ ' is given in red as 0.253 . This applies to every length or time, so therfore make its possible to predict results.

To check that this is correct, I shall apply it to an answer, which I already know.

I shall work out the time for a 60 cm pendulum as follows: -

L? tï $i^{1 / 2}$
$L=k t i ̈ i^{1 / 2}$
$0.6=0.253 *$ tï $^{1} 1 / 2$
0. 6 ?????????? tï¿½
tï¿¹/2 $=2.372$ seconds
$t=1.54$ seconds

This is the theoretical answer according to my results. Galileo's formula gave an answer of 1.555 seconds. I had a time of 1.551 seconds in my results. As
all three answers are very close, I can presume that both the direct proportion method and the Galileo methods is correct.

It would be possible to work out the time of each oscillation for a 50 metre pendulum and would be worked out as follows: -

L? tïi i $1 / 2$
$L=k t i ̈ i^{1 / 2}$
$50=0.253 * t i ̈ i^{1 / 2}$

50 ? $0.253=t i ̈ i^{1 / 2}$
tï $i^{1 / 2}=197.628$ seconds
$t=14.058$ seconds

Therefore from this formula I would be able to predict the approximate time of each oscillation of a 50 metre pendulum. By the Galileo method, one would find an answer of 14. 192 seconds.

I have drawn a graph on a separate sheet (Figure 1), which displays how length is directly proportional to time squared. I have drawn a line of best fit; crosses, which are directly on the line, are the most accurate times. Those crosses, which are furthest away from the line, are the least accurate results. From this line of best fit, it would be possible to estimate the correct time for a certain length (or visa versa).

[^1]This table of results shows that my experiments investigating the effect of mass on a pendulum also had good results. The theoretical answer for this investigation (Where the pendulum is 50 cm long and is released each time at an angle of $30 i{ }_{i}{ }^{1 / 2}$. The theoretical answer is 1.419 seconds, whereas I achieved an average of 1.379 seconds. Each time the results did not fluctuate that much because theoreretically, mass does not affect the oscillation of a pendulum. I shall now draw a graph, which shows how mass is related to time of oscillation.

It is difficult to plot a graph, which shows how one value stays precisely the same, so this graph shows how the times varied each time the experiment was repeated. The average comes to 1.379 . Before, it was possible to work out the direct proportion, however, in this experiment; this is inappropriate because for proportion to work, both figures used must change. From this information, I can ascertain that mass is not connected to time.

## * Angle of Release

Shown above is a table of results and a scatter-graph to show these results. The table of results shows that as the angle of release increases, so does the time for each oscillation, so the speed at which the pendulum swings decreases as the angle of release is increased. Theoretically, the answer should remain the same at whatever angle because a) the pendulum has a greater speed to travel as the release angle increases, but there is an increased distance to travel so the time should remain the same. And b) in Galileo's formula, it is stated that only length of pendulum and rate of gravitational acceleration is the criteria that affects the rate of oscillation, no other factor affects this whatsoever.

The scatter-graph shows how as the angle of release increases, so does the time needed to make each oscillation. The graph must not be misinterpreted, as the graph goes up, this indicates that the pendulum is slowing down. The pendulum is not speeding up. I must now theorise as to why the pendulum slows down instead of staying at the same speed throughout.

I believe that as the angle of release increases so does the height at which the pendulum drops and therefore the speed. As the speed increases, so does friction between the pendulum and air particles. More energy is needed to move past the air particles resulting in a higher loss of energy due to kinetic energy being converted into heat energy. Consequently, the pendulum will slow down at a greater rate as the angle of release is increased. This forms a negative correlation. As the angle of release is increased, the speed of the pendulum oscillation decreases.

See separate sheet (figure 2) to see how this happens. The graph has the time section in reverse to what it normally is to show that the results of this experiment have a negative correlation. It was not possible to draw a line of best fit for my results even though there was a certain negative correlation. I believe that there is a certain angle at which the oscillation will slow down, which is probably $40 i ̈ ¿^{112}$. After this point on the graph, the time slow down whereas before this they remain more-or-less constant.

I can safely say that my hypothesis regarding mass was proven to be correct. I said that mass should have no affect whatsoever on the rate of oscillation. It was possible to prove this from my results as I stated that if mass was to change, it would have no effect on the times. Gravitational acceleration is the only factor that decides how fast an object may fall in a vacuum. Pendulums with large masses should therefore be no slower or quicker than pendulums with small masses. The data that I collected was not brilliant because the results digressed to such large extents in both positive and negative directions, therefore they could be classed as inconclusive. But as the line on the graph showed, there was no pattern as to how the results altered. They also remained fairly close to the theoretical answer, so I can conclude that if the experiment had been repeated many times, the results could have been more accurate and would not have been so indifferent to the theoretical results.

Additionally, the only way that mass can affect time of oscillation is by air resistance. That is, if the size of the mass alters each time. When the mass was altered in my experiments, the size of the object changed along with its aerodynamics. This therefore theoretically slowed the object down. But, in my results, this is not shown because if air resistance was a big factor, then the results at the beginning would be quicker. However, the beginning (lower masses) actually has some of the slowest results. Whereas the higher masses did not have such slow times, they were not however the fastest. From this I can conclude that air resistance played no part in the varied times that I received. It must have been too slow for air resistance to make a significant difference.

The results of my angle of release experiments were less conclusive. However it is possible to theorise as to why the results came out this way. In my hypothesis, I was unsure as to whether angle of release actually made any difference to the time of oscillation or not. I predicted that it would probably not as air resistance increases as speed increases, therefore slowing the pendulum down. The pendulum also travels a greater distance where the angle of release is bigger, so the extra distance compensates for the extra speed.

My results could explain air resistance. As the angle of release increases, the time of each oscillation increases - it slows down. I believe that air resistance was the main factor, which influenced this result. There was a clear pattern emerge. As the angle increases the time increased in an almost predictive pattern. However, this pattern did not emerge until after 40 ï $i^{1 / 2}$. This indicates that if air resistance was the main contributing factor, then there must be a critical speed for it to reach before it will be affected by air resistance enough. The results before this virtually remained the same and alter from both positive to negative at all ends of the scale between $0 \ddot{i} ¿^{1 / 2}$ and $40 \mathrm{i}^{1 / 2}$. This is enough conclusive evidence to say that when not in a vacuum, $40 \ddot{i}^{1 / 2}$ is the approximate maximum for the results not to be influenced by air resistance.

This is experiment would have been best performed where there is no air - a vacuum. However, when I observed the angle of release experiment, I noticed that after about $45 i{ }^{\circ} ¿^{112}$, the pendulum appeared to ' flop'. I believe this was due to the speed needed to keep the pendulum string taught. Enough speed is needed to keep the string from ' bending.' This factor may
have contributed to some loss of energy in the pendulum and therefore some of the speed would had been lost. This could be solved by a pendulum which does not bend, it cannot ' flop' whilst it swings.

Therefore, I belive that angle of release does not contribute to the time of oscillation, but air resistance does. So, if this experiment could be retested in a vacuum, we would see that the times would be tighter and more accurate. Only that way could we say if angle of release affects time of oscillation.

Finally, length certainly does affect the time oscillation. I have proven this with two different ways. Firstly, my results follow a distinct pattern. There is a positive correlation between time of oscilation and mass. When Galileo's theoretical results are compared to my results, they are distinctively similar. On the graph, it is clear that my results are very accurate because most of them line up almost directly to Galileo's theoretical answer. There are absolutely no anomalous results, and any small discrepencies can only put down to human inaccuracy of timing. This is also helped by the fact that the experiment was a very fair test. The pendulum mass was the same each time and the angle of release was identical each time.

Secondly, I found that length is directly proportional to time squared. When this is appied to each of the results, they corresponded accordingly. The idea of this is that the perfect time is found from the length and a constant and then that result is compared to my actual time. I gave an example as to how effective this was where the formula gave an answer of 1.54 seconds, and I got a result of 1.551 seconds. Theoretically, I should have got a result of 1 .

555 seconds. What this proves is that length is directly proportional to time squared and that my results were also very accurate.

Evaluation

My most favourable results were the length of pendulum results. These were very good as they had a high degree of accuracy. The graph that I produced that compares my results to the theoretical results proves this. The difference between them was minimal. This shows that my methods were highly accurate in achieving those reults. However, there could have been a degree of chance in getting the timings so accurate. I did not expect these results to be so accurate.

I did not like my angle of release results or my mass results. These were not as expected, but this may have been down to timing inaccuracies. The way in which the angle of release experiments changed were justifyable because I could hypothesise as to how the results came to do that. In the end, I believe that I came up with a fairly good theory for the way in which the results behaved. I do not believe that it is possible to expalin the mass experiments' results. These were anomalous because of the way that they changed so much with no particular pattern. I suspect that as the other experiments were faily accurate in their timing, it could be due to the method of the experiment itself - perhaps it was not a fair test. My end conclusion did not correspond with the actual results but did correspond with my hypothesis. Therefore, I have no evidence to back up my conclusion, even though I believe it is the right conclusion.

In my experiments, if I timed an experiment wrong, and I was sure that I had significantly mis-calculated it, then I repeated that part of the experiment until I was sure that I had timed it to the highest level of accuracy. This is the only way that accuracy can be improved without repeating the experiment many times to get an average score.

The only way that the experiments can be improved is their timing methods and the conditions under which the experiments are performed. The timing was completely down to human judgement. There would always be a degree of inaccuracy, as it takes time for one's brain to process when the pendulum reaches its peak and to send a message along the nerves to the fingers so that the stopwatch can be stopped. This is a factor, which will cost split seconds and will reduce inaccuracy. The only way that this can be improved is by estimating when the pendulum will be at its peak height so that the stopwatch can be stopped at the correct moment. Even the accuracy of the stopwatch could be questioned. I timed ten oscillations with accuracy to two decimal places, then by dividing by ten I got three decimal places. Ideally, it would be timed with more decimal places, with an atomic clock. These are reputedly the most accurate clocks ever, boasting accuracy of ?? 1 second over hundreds of thousands of years.

Furthermore, accuracy would be improved if the experiments were repeated many times until an average was reached whereby it did not fluctuate at all. Human beings are also not accurate to tell when the experiment starts or finishes. What would be nedded is a sensor, which could detect when the experiment had started and when it finishes (i. e. its peak height). This could trigger an electronic signal to highly accurate timing equipment. This however would be very difficult and very expensive to set up. There would be little point in doing so especially as the theories have been tested many times before and proven to be correct.

Very importantly, though is that the Galileo formula is only theoretical for a vacuum and does not take into account that there is air resistance. Ideally, it would be best to perform the experiment in an environment where there is no air (a vacuum). This way, it would be certain that air resistance could have no affect whatsoever on the results, and it could then be called a fair test. Galileo's formula has only been tested out on this planet where the gravitational acceleration is 9.8 . This could be tested elsewhere, where there is differing gravitational acceleration.

These are the only ways in which I believe that additional evidence can be found which would support my investigation. However the measures needed to check them are not necessary. The only reallistic way in which the investigation can be backed up is by repeating the experiment until the average remains at a constant, and the experiment is repeated whilst in a vacuum with more accurate timing equipment.


[^0]:    * Length of pendulum

[^1]:    * Mass of Pendulum

