Cusum methodology for dealing censored data engineering essay

Engineering

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1. INTRODUCTION

The products have been designed with high reliabilities because of upcoming requirements of the costumers from the manufacturing technologies/industries. We can save a lot of time and cost by using highly censored data collected from life time’s distribution. An important issue in life-testing applications for industrial engineering is how to develop a control chart for monitoring the mean level life time of products under censoring. In many cases when life data are analyzed, total number of the units in the sample may not have failed or the exact times-to-failure of all the units are not known. This type of data is commonly called censored data. Many techniques have been discovered in the past to overcome the deficiency caused by the presence of censored units in the data. Conditional expected value control chart (CEV) using different probability distribution like Lu and Tsai (2008) has done work on type- I censored data using Gamma distribution and proposed EWMA CEV control chart for monitoring mean level of the gamma life times. Zhang and Chen (2004) developed low-sided and an upper-sided EWMA CEV control chart to detect the mean level lifetime shifts in ascending and in descending pattern for the Weibull process. These control charts developed for skewed distribution can only detect one-side shift of the mean lifetime level at a time. Steiner and Mackay (2000) developed a charting method based on the conditional expected values (CEVs) which allows quick detection of corrosion in the process quality with highly censored data under normality. Steiner and Mackay (2001a) also developed Shewhart type control chart based on CEVs. It can monitor process mean when the censoring occurs at a fixed level based. Tsai and Lin (2009) proposed a EWMA control chart based on Gompertz lifetimes to detect mean level shifts in decreasing or in larger increase with type I censoring. Olteanu (2010) considered the Cumulative Sum Control Charts for Censored data and given a comparison of the censored CUSUM control chart with EWMA censored chart.

2. MATERIALS AND METHODS

2. 1 CUSUM FOR CENSORED DATA

We construct the CUSUM (cumulative sum) chart to monitor shifts in the scale parameter using Rayleigh lifetimes. We assume that in-control values of the scale parameter are known and define an out-of-control situation characterized by =, where = (1-q) , where q*(100%) denotes a percent change in. We generate samples of " n" items in a testing process and T represents the lifetime of the products. We stop the test at a predetermined time " C". The products not failing by time " C" produces a censored lifetime. There is a random number of censored in each sample i, with values replaced by the censoring time " C". The test statistics of the traditional CUSUM control chart are transformed using standard exponential transformation and are represented as: For , Where i= 1, 2, 3 . . ., m, and " q" shows amount of shift change and " m" shows the subgroup size. When the CUSUM chart signals, then it shows the chart indicates a decrease in the scale parameter. Similarly if we want to detect a positive shift in the scale parameter, given by q <0, then we getWe use the chart statistics as: Where i= 1, 2, 3 . . ., m, and q shows amount of shift change. (Where the value of ei is derived using formula from Olteanu (2010) on CUSUM control charts for censored reliability data.)When the chart signals it indicates an increase in the scale parameter.(where h- , h+ shows threshold values and & are the test statistics of CUSUM chart). The stimulation study is designed as: Generate samples of size n= 3, 5, 10, as come across in similar papers and in quality control sampling practice. If the lifetime random variable T~Rayleigh (t;) distribution with Type-I right censoring at a predetermined time C, then Pc= 1-F(t;), where F(t;) is the cumulative density function of Raleigh distribution where is the scale parameter. For example, a 50% censoring rate shows that approximately half of the products fail by the stopping time C. Therefore half of the units are being censored. We have generated data from Rayleigh distribution correspond to theoretical censoring rates Pc= 5%, 30%, 60%, 80% i. e. we have evaluated the performance of the charts in low, moderate and high censoring situations. The theoretical censoring rate corresponds to an in-control situation whereas the actual numbers of censored units in generated random sample are random. With the decrease in the process mean or characteristic life it is expected to have a lower number of censored units in the sample. We designed the chart for corresponding theoretical in-control censoring rate. In-control value for the scale parameter is equal to 1. 11, without loss of generality. For the negative shift size q= 2. 5%, 5%, 10%, 20% 30% could be used. The out-of-control value is hence = (1-q) . We assessed situations with small, moderate and large shifts. Desired in-control run length IARL = 370, as frequently used in quality control procedures. We compared the CUSUM chart to the CEV EWMA with smoothing parameters λ = 0. 05, 0. 1, 0. 2

3. DESCRIPTION OF SIMULATION ALGORITHM

We evaluated the performance of the likelihood ratio based CUSUM chart and compared it to the performance of the CEV EWMA chart in the above listed circumstances using Monte Carlo simulations. The steps required for computation are as: Prepare the situation parameters with the desired values: the in-control scale= = 0. 5, the sample size n, the censoring proportion Pc, and the shift of interest q. Based on the values and Pc determined the censoring time C as determined, specify the threshold values i. e. h- and h+ and calculate the out-of-control values of the parameter = *(1-q). Now search the threshold space, using the bisection search method. For each transitional threshold perform 5, 000 replications of the simulation. At each simulation step generate samples of n observations from a Rayleigh distribution with parameter, as they are censored according to Type I censoring scheme, calculate the CUSUM statistics, Ci, and obtain an in-control run length for this simulation. From average over the 5, 000 run lengths calculate the in-control ARL (ARLo) and we calculated a standard error of the run lengths. Now check if the estimated ARLo was close to the desired in-control ARL of 370.

3. 1 PERFORMANCE OF CUSUM CHART AND ITS PROPERTIES

In this section we discuss the relative performance of the CUSUM and CEV EWMA charts, and the trends in CUSUM chart performance depending on censoring rate, shift size and value of the in-control scale parameter. The results are being calculated for n= 5 with 10%and 30% decrease in scale parameter. We evaluate the charts' performance in terms of the out-of-control average run length, while the charts are designed to provide an in-control average length of 370. The best chart gives the lowest out-of-control ARL (ARL1). Table 1: Comparison of Out-Of-Control ARL (ARL1) = 1, h= 5 and n= 5Chart typePc→ 0. 050. 5

0. 8

EWMA 0. 057599148EWMA 0. 1100103177EWMA 0. 2134160202

CUSUM

697377The table 1 shows the comparison of ARL1 of EWMA (0. 05, 0. 1, 0. 2) and CUSUM for 10% decrease in scale parameter (= 1 and n= 5). The results show that CUSUM ARL1 is least among all compared charts. So CUSUM shows most efficient results in the presence of censored data. The results described above are graphically represented in figure1. Comparison between

Out-of-Control ARL to detect 10% decrease in the Scale-parameter with the censored CUSUM Chart, and with the CEV EWMA with Smoothing Parameter λ = 0. 05; 0. 1, and 0. 2. Figure 1 CUSUM and EWMA ComparisonThe Figure 1 show that CUSUM censoring method performs better than EWMA 0. 05, EWMA 0. 1 and EWMA 0. 2. It can be seen from the lines showing ARL1 values in the graph that CUSUM Chart for censoring is performing better than EWMA method so we can conclude that the CUSUM censoring chart can detect assignable causes more rapidly than EWMA censoring chart. Similarly the Figure 4. 2 below shows (for = 0. 8 and n= 5) EWMA 0. 05, EWMA 0. 1, EWMA 0. 2 performs approximately the same and shows same out of control ARLs still CUSUM performance is better based on discussed properties. Table 2: Comparison of Out-Of-Control ARL (ARL1) for = 0. 8, h= 4 and n= 5Chart typePc→ 0. 050. 5

0. 8

EWMA 0. 051850200EWMA 0. 11851184EWMA 0. 21860185

CUSUM

182022The table 2 shows the comparison of ARL1 of EWMA (0. 05, 0. 1, and 0. 2) and CUSUM charts in the presence of Type I Censoring. The results show that CUSUM ARL1 is least among all compared charts. So CUSUM shows most efficient results in the presence of censored data. The results described above are graphically represented in figure2. Comparison between Out-of-Control ARL’s, 30% decrease in the Scale Parameter with the censored CUSUM Chart, and with the CEV EWMA with Smoothing Parameter λ = 0. 05; 0. 1, 0. 2 and = 0. 8, h= 4 and n= 5. Figure 2 CUSUM and EWMA

Relative PerformanceTable 3 shows censoring rate, fixed threshold values, in control ARL out of control ARL of Censored CUSUM and CEV EWMA control charts for Rayleigh distribution with parameter = 0. 5 and sample size n= 5. Table 3: ARL for CUSUM and EWMA Chart based on Rayleigh DistributionPcThresholdARLoARL1ARLo

ARL1

CUSUMEWMA0. 32. 72370. 8552. 3637790. 70. 52. 51370. 7259. 9837191

0. 6

2. 23370. 5562. 5436896From the table3, we can see that the CUSUM censoring chart performance better for Rayleigh distribution than EWMA CEV control chart because ARL1 for CUSUM control chart is less than ARL1 EWMA CEV control chart. Table 4: ARL for CUSUM Chart based on Rayleigh Distribution for different values of n. PcThresholdARLoARL1For n= 3ARL1For n= 5

ARL1

For n= 7

0. 32. 72370. 8566. 5652. 3649. 540. 52. 51370. 7270. 2559. 9853. 70

0. 6

2. 23370. 5574. 3362. 5457. 66Table 4 shows censoring rates and fixed threshold values, in control ARL , out of control ARL of Censored CUSUM control charts for Rayleigh distribution with parameter = 0. 5 and sample sizes n= 3, 5, 7. It can be seen with the increase in sample size censored CUSUM control charts for Rayleigh distribution detects the shift more rapidly.

4. Max-CUSUM Control Chart in the presence of censored data

We construct the Max-CUSUM (cumulative sum) chart to monitor process mean level shifts as well as standard deviation "" shifts. We assume that in-control values of the scale parameter are known and define an out-of-control situation characterized by = 1, where 1 = (1-q) O , where q*(100%) denotes a percent change in. We generate samples of " n" items in a testing process and T represents the lifetime of the products. We stop the test at a predetermined time " C". The products not failing by time " C" produces a censored lifetime. There is a random number of censored in each sample i, with values replaced by the censoring time " C". The test statistics of the Max- CUSUM control chart are transformed using standard exponential transformation and are represented as: For,

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Where i= 1, 2, 3 . . ., m, and " q" shows amount of shift change and " m" shows the subgroup size. When the Max-CUSUM chart signals, then it shows the chart indicates a decrease in the process mean level and scale parameter. Similarly if we want to detect a positive shift in the scale parameter, given by q <0, then we getWe use the chart statistics as: Where i= 1, 2, 3 . . ., m, and q shows amount of shift change. Xij and Zij follow standard normal distribution. Now the test statistics to be used is: Mi = Max [ ]When the chart signals it indicates an increase in the mean level or Scale parameter . When the chart signals it indicates a decrease in the mean level or Scale parameter . The stimulation study is designed as: Generate samples of size n= 3, 5, 10, as come across in similar papers and in quality control sampling practice. If the lifetime random variable T~Rayleigh (t;) distribution with Type-I right censoring at a predetermined time C, then Pc= 1-F (t;), where F(t;) is the cumulative density function of Raleigh distribution where is the scale parameter. For example, a 50% censoring rate shows that approximately half of the products fail by the stopping time C. Therefore half of the units are being censored. We have generated data from Rayleigh distribution correspond to theoretical censoring rates Pc= 5%, 30%, 60%, 80% i. e. we have evaluated the performance of the charts in low, moderate and high censoring situations. The theoretical censoring rate corresponds to an in-control situation whereas the actual numbers of censored units in generated random sample are random. With the decrease in the process mean or characteristic life it is expected to have a lower number of censored units in the sample. Desired in-control run length IARL = 370, as frequently used in quality control procedures. We compared the CUSUM chart to the Max-CUSUM with smoothing parameters λ = 0. 1Table 5 shows censoring rate, fixed threshold values, in control ARL out of control ARL of Censored CUSUM and CEV EWMA control charts for Rayleigh distribution with parameter = 0. 5 and sample size n= 5. Table 5: ARL for CUSUM and EWMA Chart based on Rayleigh DistributionPcThresholdARLoARL1ARLo

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5. CONCLUSIONS

In this paper a new method is developed to deal with censored data for Rayleigh distribution. The method developed is entitled as CUSUM method for dealing censored data'. This method uses the methodology of Cumulative sum control charts but as it is being used in the presence of censored data so the control limits and methodology is being updated and transformed to deal with censored data. The numerical results developed in this paper for CUSUM method is also compared with CEV control charting methodology results showing CUSUM censoring methodology is more efficient in the presence of censored data for Rayleigh distribution. In comparison to CUSUM charts, Max CUSUM chart performs better in detecting the shift in the parameters. In future the work could be extended for multivariate distributions.