# Concept of surface area of a cube 

The Concept of Surface Area of a Cube Introducing this concept to in grades 5 and 6 BACKGROUND INFORMATION FOR TASK The is to build alarge sportsequipment bin. The finished dimensions will be 1 cubic yard ( $3 \mathrm{ft} . \times 3 \mathrm{ft} . x$ 3ft.). The top will hinge at the back to open, and it will latch closed at the front. Once finished, the all exterior faces will be painted to help waterproof it. All materials have been donated except the paint. The class must raise money to purchase it, so they must be economical.

MATERIALS

1. 3 sheets of plywood, each $3 \mathrm{ft} . \times 6 \mathrm{ft}$.
2. Hinges, a latch, and screws to attach them
3. Epoxy to join pieces
4. Paint to be bought
5. A classroom volunteer has all the necessary tools and knowledge of construction.

NOTES AND OBSERVATIONS

1. The volunteer explains that if it is necessary to cut the plywood, it will take 5 minutes for each saw cut.
2. To construct the bin, the volunteer must apply the epoxy to 2 edges to make a seam, then brace them together to dry for 15 minutes. After that, 2 more edges can be joined, etc. When the bottom is joined to the sides, all four edges can be glued at the same time. That is, the bottom counts as one seam. After the bottom and sides are assembled, the top will be attached.
3. It will take 20 minutes to put on the hinges and latch.
4. A gallon of paint covers 150 sq. ft. and costs . 00 (tax-free for schools)
5. A quart of paint covers 40 sq. ft. and costs . 00 (tax-free for schools)

TASK ACCOMPLISHMENT
(How to introduce the concept of surface area of a cube to students in grades 5 and 6)

Part 1 - The prerequisite skills necessary to complete this problem; Guiding the students so that they can calculate how many sheets of plywood must be used and determine how many solid seams must be made

The prerequisites for this task include recognition of the cube and knowledge of its properties, and the concept of surface area in general. This in turn expects that the students have previously dealt with the concept of the square and flat area in earlier classes so that they can now grasp the concept of the surface area of the cube. The skills required are the ability to measure in feet, derive formulae, and perform simple addition and multiplication calculations related to working out areas, the time taken to complete a project, and the cost of materials. Besides this is the simple ability to compare quantities required at the final stage. They must also be familiar with the concept of 3 dimensions as found in objects around us; that there is a third measure of depth/height besides length and breadth of two dimensional objects and drawings. And, appreciate how we can use (apparently) two-dimensional materials (the plywood sheets) to construct three-dimensional objects (the cube shaped bin).

The detailed sketches below will greatly assist the students in easily recognizing and carrying out the task and require only a little further explanation.

Students should recognize that the number of pieces that need to be cut corresponds to the six faces of the cube. To account for the thickness of the sheets and allow the seams of edges to be made, four of these parts will have shorter dimensions than the 3 ft . $\times 3 \mathrm{ft}$. dimensions of the first two
parts (from the first sheet for the top and bottom sides). Given the information contained in part 2 of the notes, the students should be aware how only 5 seams need to be made. The cube has 12 edges in total and two meet to form a seam except that in some cases more than two edges can be affixed at the same time. It should be pointed out that when the four front, back, left side and right side parts are joined, the last piece can be affixed at both ends simultaneously, so only a total of 3 seams need to be made at this stage. The next two seams will be made when attaching the bottom and top sides. When the construction of the bin is completed (i. e. before being painted and the latch and hinges attached) it should look somewhat like this:

Part 2 - Guiding the students so that they can calculate total construction time (including cutting the wood)

The pertinent information for calculating the total construction time is the first three parts of the notes. To summarize, these are 5 minutes for each saw cut of the plywood sheets, 15 minutes for bracing edges together to let the seam dry, and 20 minutes to put on the hinges and latch. The students only need to determine how many times each of these subtasks need to repeated; recognize that we then need to multiply these figures with the aforementioned unit times, and to finally sum the three resulting quantities. This will give us the total construction time prior to painting the bin. As the figure will be in minutes, the students also need to be able to interpret this in hours and minutes for convenience. The calculation the students should be making to arrive at the correct estimation is given below.

Total amount of time taken for construction
$=$ Sum of the amounts of time taken for each of the three set of subtasks
$=(5 \times$ no. of saw cuts $)+(15 \times$ bracing edges $)+(20 \times$ putting on hinges and latch)
$=(5 \times 6)+(15 \times 5)+(20 \times 1)$
$=30+75+20$
$=125$ minutes
$=2 \mathrm{hrs} 5 \mathrm{mins}$
Part 3 - How the students will arrive at exterior surface area
To arrive at the figure for external surface area, the students should be made aware that it is the total exterior/outside exposed areas of the wood whose area we wish to calculate. Also, it will be necessary to remind the students that all 6 faces of the cube are of equal square shape. If necessary, they should be prompted to point this out themselves as being necessary to work out the total exterior surface area, and that the whole area is made up of 6 equal parts. Furthermore, that each face has the same dimensions, therefore all faces have the same area obtained by multiplying the two dimensions (or squaring either since they too are equal). This means, we only need to work out the area of one face to save our time calculating the total surface area. In our case, the dimensions of each face are $3 \mathrm{ft} \times 3 \mathrm{ft}$. So, we need to multiply either 3 by 3 , or calculate 3 squared (32). Both will yield the same answer of 9 square feet. Now that we know the area of one face, we can simply add this figure to itself 6 times for each of the faces, or just multiply the figure by 6 to get the answer for total surface area of 54 square feet. Students who are capable of forming formulae, may be encouraged to derive the following:
exterior surface area $=6 \times$ area of each face (length $x$ breadth)
Part 4 - Guiding the students so that they can decide how much paint they
will need and how much it will cost
To decide how much paint the students will need to use to paint the exterior surface area of the cube shaped bin, they will use the figure calculated for exterior surface area ( 54 sq. ft.), and find out how many tins of paint of each of the two available types would be sufficient. This will also enable us to know how much the paint supply will cost for each option. The students will then be able to compare both and identify which option is most economical based on the lowest cost figure. In our case, one gallon of paint can cover 150 sq. ft., which is more than sufficient to paint even two such bins. On the other hand, a quart of paint can only cover 40 sq. ft., which the students should be able to recognize as being insufficient and that we would therefore require two such tins. Now that the quantities required of each option are known, we can multiply each by their respective unit cost figures (i. e. of one tin) to work out the total cost. It should be pointed out that the unit cost figure to be used in the calculation is the cost of one tin only, and that it is the gross cost figure i. e. without the added tax. This is because paint is available tax-free for schools, though students should be reminded nonetheless to use the paint providently Students will be working out the following (and then comparing the two figures to see which is the lowest):

Option 1 (buy a gallon): Total cost $=1 \times$ cost of one gallon
Option 2 (buy two quarts): Total cost $=2 \times$ cost of one quart

