The mysterious series 3 problems

The Mysterious Series Dan and John are trying to compute the infinite sum 1

1 1 + … Dan writes 1 1 1) + … = 0 = 0 = 0

and concludes that the sum of the series is 0. John writes:

1+ (– 1 + 1) + (-1 + 1) + (-1 + 1) +…

= 0 = 0 = 0

and concludes that sum must be equal to 1. Who do you think is right?

ANSWER:

Both answers may seem correct, but it does not seem to approach any convergent number. There is however a third answer : Let S be the sum of the infinite series 1-1+1-1+1… hence 1-S= 1-(1-1+1-1+1…). But since 1-S is also S, thus 1-S= S. Algebraically manipulated, 1= 2S and S= 1/2.

The truth is the series has no sum in the strictest sense, but if it had a sum, it must ½.

2. A company is making shoeboxes from cardboard. The cardboard is 20 inches in length and 16 inches in width. The company is going to cut square pieces off each corner as shown in the diagram below and fold the sides up. Explain how you can find the formula for the volume of the box in terms of x. Show all of your work .

ANSWER:

The volume of the box to be made is easily taken by the formula, length X width X height. The length X width is simply the area of the inner box diagramed above. The height is simply x. Thus the expression for the volume of the box is the following:

Volume = (16-2x)(20-2x)(X)

Simplifying we obtain the expression 320x-72x2-4x3.

3. Billy bob has 200 yards of fencing material, and he wishes to build a rectangular kennel with four sections, as shown below. Find the dimensions of the individual pens if the total enclosed area is to be as large as possible? Prove that you have accurately determined the total maximum area.

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ANSWER:

Let x be the width of each individual pen. If the total fencing is 200 yards, the resulting length of the whole pen would be (200-5x)/2 and the area of the whole pen would be [(200-5x)/2](x). The formula for the area of the whole pen would be:

Area= 100x-(5/2)x2

To maximize the area, derive the formula for the area, equate to zero and then solve for x. The derivative would then be 100-5x= 0, thus x= 20. Substituting back into the equation, the length of the whole pen would be 50 yards and the width of the pen would be 20 yards. If individual pens were to be measured, the length would be divided by 4, thus the length of each individual pen would be 12. 5 yards. Since the width of each pen is 20 yards, each pen would have an area of 250 square yards.