Chapter 5 interest rates 5-1

Business

Your bank is offering you an account that will pay 20% interest in total for a two-year deposit. Determine the equivalent discount rate for a period length of:

- a. Six months.
- b. One year.
- c. One month.

For 6 months is of 2 years, using our rule. So the equivalent 6-month rate is 66%. For one year is half of 2 years. So the equivalent 1-year rate is 54%. For one month is of 2 years, using our rule. So the equivalent 1-month rate is 0. 63%. 5-2.

Which do you prefer:

- a. bank account that pays 5% per year (EAR) for three years or An account that pays 2 every six months for three years?
- b. An account that pays 7 every 18 months for three years?
- c. An account that pays per month for three years? If you deposit $1 into a bank account that pays 5% per year for 3 years you will have after 3 years.
- d. If the account pays per 6 months then you will have after 3 years, so you prefer every 6 months.

If the account pays per 18 months then you will have after 3 years, so you prefer 5% per year. If the account pays per month then you will have after 3 years, so you prefer every month. 5-3. Manyacademicinstitutions offer a sabbatical policy. Every seven years a professor is given a year free of teaching and other administrative responsibilities at full pay. For a professor

earning $70, 000 per year who works for a total of 42 years, what is the present value of the amount she will earn while on sabbatical if the interest rate is 6% (EAR)? Thus, your remaining balance is $20, 092. 39. If you prepay an extra $100 today, you will lower your remaining balance to $20, 092. 39 – 100 = $19, 992. 39. Though your balance is reduced, your required monthly payment does not change. Instead, you will pay off the loan faster; that is, it will reduce the payments you need to make at the very end of the loan. How much smaller will the final payment be? With the extra payment, the timeline changes: That is, we will pay off by paying $500 per month for 47 months, and some smaller amount, $500 – X, in the last month.

To solve for X, recall that the PV of the remaining cash flows equals the outstanding balance when the loan interest rate is used as the discount rate: Solving for X gives. So the final payment will be lower by $143. 14. You can also use the annuity spreadsheet to determine this solution. If you prepay $100 today and make payments of $500 for 48 months, then your final balance at the end will be a credit. The extra payment effectively lets us exchange $100 today for $143. 14 in four years.

We claimed that the return on this investment should be the loan interest rate. Let's see if this is the case: so it is. Thus, you earn a 9% APR (the rate on the loan). 5-19. Consider again the setting of Problem 18. Now that you realize your best investment is to prepay your student loan, you decide to prepay as much as you can each month. Looking at your budget, you can afford to pay an extra $250 per month in addition to your required monthly payments of $500, r $750 in total each month. How long will it take you to pay off the loan? The timeline, in this case, is: and we want to determine the

number of monthly payments N that we will need to make. That is, we need to determine what length annuity with a monthly payment of $750 has the same present value as the loan balance, using the loan interest rate as the discount rate. As we did in Chapter 4, we set the outstanding balance equal to the present value of the loan payments and solve for N. We can also use the annuity spreadsheet to solve for So, by prepaying the loan, we will pay off the loan in about 30 months or 2? years, rather than the four years originally scheduled. Because of N of 30. 02 is larger than 30, we could either increase the 30th payment by a small amount or make a very small 31st payment. We can use the annuity spreadsheet to determine the remaining balance after 30 payments.

N	I	PV	PMT	FV
30	0. 75 %	20, 092. 39	- 750	- 13. 86

If we make a final payment of $750. 00 + $13. 86 = $763. 86, the loan will be paid off in 30 months. 5-20. Oppenheimer Bank is offering a 30-year mortgage with an APR of 5. 25%. With this mortgage, your monthly payments would be $2000 per month. In addition, Oppenheimer Bank offers you the following deal: Instead of making the monthly payment of $2000 every month, you can make half the payment every two weeks, so that you will make 52 2 = 26 payments per year. With this plan, how long will it take

to pay off the mortgage of $150, 000 if the EAR of the loan is unchanged? If we make every 2 weeks the timeline is as follows. Using the formula for the loan payment. Next, we write out the cash flows with the extra payment. To determine the outstanding balance we discount at the original rate, i. e. Next, we calculate the loan payment on the new mortgage. You have a credit card debt of $25, 000 that has an APR (monthly compounding) of 15%. Each month you pay the minimum monthly payment only. You are required to pay only the outstanding interest. You have received an offer in the mail for an otherwise identical credit card with an APR of 12%. After considering all your alternatives, you decide to switch cards, roll over the outstanding balance on the old card into the new card, and borrow additionalmoneyas well.

How much can you borrow today on the new card without changing the minimum monthly payment you will be required to pay? The discount rate on the original card is: Assuming that your current monthly payment is the interest that accrues. The new discount rate is So, So by switching credit cards you are able to spend an extra. You do not have to pay taxes on this amount of new borrowing, so this is your after-tax benefit of switching cards. 5-25. In 1975, interest rates were 7. 85% and the rate of inflation was 12. 3% in the United States. What was the real interest rate in 1975?

How would the purchasing power of your savings have changed over the year? The purchasing power of your savings declined by 3. 96% over the year. 5-26. If the rate of inflation is 5%, what nominal interest rate is necessary for you to earn a 3% real interest rate on your investment? implies. Therefore, a nominal rate of 8. 15% is required. 5-27. Can the nominal interest rate available to an investor be significantly negative? Hint:

Consider the interest rate earned from saving cash " under the mattress. Can the real interest rate be negative? Explain. By holding cash, an investor earns a nominal interest rate of 0%.

Since an investor can always earn at least 0%, the nominal interest rate cannot be negative. The real interest rate can be negative, however. It is negative whenever the rate of inflation exceeds the nominal interest rate. 5-28. Consider a project that requires an initial investment of $100, 000 and will produce a single cash flow of $150, 000 in five years.

- a. What is the NPV of this project if the five-year interest rate is 5% (EAR)?
- b. What is the NPV of this project if the five-year interest rate is 10% (EAR)? c. What is the highest five-year interest rate such that this project is still profitable?

NPV = –100, 000 + 150, 000 / 1. 055 = $17, 529. b. NPV = –100, 000 + 150, 000 / 1. 105 = –$6862.

- c. The answer is the IRR of the investment: IRR = (150, 000 / 100, 000)1/5 – 1 = 8. 45%. 5-29.

Suppose the term structure of risk-free interest rates is as shown below:

- a. Calculate the present value of an investment that pays $1000 in two years and $2000 in five years for certain.
- b. Calculate the present value of receiving $500 per year, with certainty, at the end of the next five years. To find the rates for the missing years in the table, linearly interpolate between the years for

which you do know the rates. (For example, the rate in year 4 would be the average of the rate in year 3 and year 5).

- c. Calculate the present value of receiving $2300 per year, with certainty, for the next 20 years. Infer rates for the missing years using linear interpolation.

Since the opportunity cost of capital is different for investments of different maturities, we must use the cost of capital associated with each cash flow as the discount rate for that cash flow. Unfortunately, we do not have a rate for 4-year cash flow, so we linearly interpolate. Since the opportunity cost of capital is different for investments of different maturities, we must use the cost of capital associated with each cash flow as the discount rate for that cash flow. Unfortunately, we do not have a rate for a number of years, so we linearly interpolate. Using the term structure in Problem 29, what is the present value of an investment that pays $100 at the end of each of years 1, 2, and 3? If you wanted to value this investment correctly using the annuity formula, which discount rate should you use?

PV = 100 / 1. 0199 + 100 / 1. 02412 + 100 / 1. 02743 =$285. 61.

To determine the single discount rate that would compute the value correctly, we solve the following for r:

PV = 285. 1 = 100/(1 + r) + 100 / (1 + r)2 + 100/(1 + r)3 = $285.

This is just an IRR calculation. Using trial and error or the annuity calculator, r = 2. 50%. Note that this rate is between the 1, 2, and 3-yr rates are given. 5-31. What is the shape of the yield curve given the term structure in Problem 29? What expectations are investors likely to have about future

interest rates? The yield curve is increasing. This is often a sign that investors expect interest rates to rise in the future. 5-32. Suppose the current one-year interest rate is 6%.

One year from now, you believe the economy will start to slow and the one-year interest rate will fall to 5%. In two years, you expect the economy to be in the midst of a recession, causing the Federal Reserve to cut interest rates drastically and the one-year interest rate to fall to 2%. The one-year interest rate will then rise to 3% the following year, and continue to rise by 1% per year until it returns to 6%, where it will remain from then on. a. If you were certain regarding these future interest rate changes, what two-year interest rate would be consistent with these expectations? What current term structure of interest rates, for terms of 1 to 10 years, would be consistent with these expectations? c. Plot the yield curve in this case. How does the one-year interest rate compare to the 10-year interest rate? a. The one-year interest rate is 6%. If rates fall next year to 5%, then if you reinvest at this rate over two years you would earn $(1.06)(1.05) = 1.113$ per dollar invested. This amount corresponds to anEAR of $(1.113)1/2 – 1 = 5.50\%$per year for two years. Thus, the two-year rate that is consistent with these expectations is 5.0%. b. We can apply the same logic for future years: c. We can plot the yield curve using the EARs in (b); note that the 10-year rate is below the 1-year rate (yield curve is inverted). 5-33. Figure 5.4 shows that Wal-Mart's five-year borrowing rate is 3.1% and GE Capital's is 10%. Which would you prefer? $500 from Wal-Mart paid today or a promise that the firm will pay you $700 in five years? Which would you choose if GE Capital offered

you the same alternatives? We can use the interest rates each company must pay on a 5-year loan as the discount rate.

PV for GE Capital = 700 / 1. 105 = $434. 64 < $500 today, so take the money now.

PV for Wal-Mart = 700 / 1. 0315 = $600. 90 > $500 today, so take the promise. 5-34.

Your best taxable investment opportunity has an EAR of 4%. Your best tax-free investment opportunity has an EAR of 3%. If your tax rate is 30%, which opportunity provides the higher after-tax interest rate? After-tax rate = 4%(1 – . 30) = 2. 8%, which is less than your tax-free investment with pays 3%. 5-35. Your uncle Fred just purchased a new boat. He brags to you about the low 7% interest rate (APR, monthly compounding) he obtained from the dealer. The rate is even lower than the rate he could have obtained on his home equity loan (8% APR, monthly compounding). If his tax rate is 25% and the interest on the home equity loan is tax-deductible, which loan is truly cheaper? The after-tax cost of home equity loan is 8%(1 – . 25) = 6%, which is cheaper than the dealer's loan (for which interest is not tax-deductible). Thus, the home equity loan is cheaper. (Note that this could also be done in terms of EARs.) 5-36.

You are enrolling in an MBA program. To pay your tuition, you can either take out a standard student loan (so the interest payments are not tax-deductible) with an EAR of 5[pic] or you can use a tax-deductible home equity loan with an APR (monthly) of 6%. You anticipate being in a very low tax bracket, so your tax rate will be only 15%. Which loan should you use?

Using the formula to convert an APR to an EAR: So the home equity loan has an EAR of 6. 168%. Now since the rate on a tax-deductible loan is a before-tax rate, we must convert this to an after-tax rate to compare it. Since the student loan has a larger after-tax rate, you are better off using the home equity loan. 5-37. Your best friend consults you for investment advice. You learn that his tax rate is 35%, and he has the following current investments and debts: A car loan with an outstanding balance of $5000 and a 4. 8% APR (monthly compounding) Credit cards with an outstanding balance of $10, 000 and a 14. 9% APR (monthly compounding). Regular savings account with a $30, 000 balance, paying a 5. 50% EAR ¦ A money market savings account with a $100, 000 balance, paying a 5. 25% APR (daily compounding) A tax-deductible home equity loan with an outstanding balance of $25, 000 and a 5. 0% APR (monthly compounding) a. Which savings account pays a higher after-tax interest rate? b. Should your friend use his savings to pay off any of his outstanding debts?

Explain.

- a. The regular savings account pays 5. 5% EAR, or 5. 5%(1 – . 35) = 3. 575% after tax. The money-market account pays (1 + 5. 25%/365)365 – 1 = 5. 39% or 5. 39%(1 – . 35) = 3. 50% after tax. Therefore, the regular savings account pays a higher rate.

- b. Your friend should pay off the credit card loans and the car loan since they have after-tax costs of 14. % APR and 4. 8% APR respectively, which exceed the rate earned on savings. The home equity loan should not be repaid, as its EAR = (1 + 5%/12)12 – 1 = 5. 12%, for an after-tax rate of only 5. 125(1 – . 35) = 3. 33%, which is

below the rate earned on savings. 5-38. Suppose you have an outstanding debt with an 8% interest rate that can be repaid anytime, and the interest rate on U. S. Treasuries is only 5%. You plan to repay your debt using any cash that you don't invest elsewhere. Until your debt is repaid, what cost of capital should you use when evaluating a new risk-free investment opportunity? Why?

The appropriate cost of capital for new risk-free investment is 8% since you could earn 8% without risk by paying off your existing loan and avoiding interest charges. 5-39. In the summer of 2008, at Heathrow Airport in London, Bestofthebest (BB), a private company, offered a lottery to win a Ferrari or 90, 000 British pounds, equivalent at the time to about $180, 000. Both the Ferrari and the money, in 100-pound notes, were on display. If the U. K. interest rate was 5% per year, and the dollar interest rate was 2% per year (EARs), how much did it cost the company in dollars each month to keep the cash on display? That is, what was the opportunity cost of keeping it on display rather than in a bank account? (Ignore taxes.) Because the prize is in pounds, we should use the pound interest rate (comparable risk). 7 pounds per month, or $733 per month at the current exchange rate. 5-40. Your firm is considering the purchase of a new office phone system. You can either pay $32, 000 now or $1000 per month for 36 months. Suppose your firm currently borrows at a rate of 6% per year (APR with monthly compounding).

Which payment plan is more attractive? Suppose your firm currently borrows at a rate of 18% per year (APR with monthly compounding).

Which payment plan would be more attractive in this case?

- a. The payments are as risky as the firm's other debt. So the opportunity cost = debt rate. PV(36 month annuity of 1000 at 6%/12 per month) = $32, 871. So pay cash.

- b. PV(annuity at 18%/12 per mo) = $27, 661.

- a. The payments are as risky as the firm's other debt. So the