C4 summary sheet

simplify algebraic fractions by factorising and cancelling factorse. g. $(3x+6)/(x^2-4) = 3(x+2)/(x+2)(x-2) = 3/(x-2)$add and subtract by finding a common denominatore. g. simplify $2y/x(x+3) + 1/y^2(x+3) +x/y$

- find the common denominator (take all of the individual bits on the bottom row and multiply them together) $= xy^2(x+3)$

-make each of the fractions have the common denominator at the bottom by multiplying the whole fraction by different individual bits

$= 2y^3/y^2x(x+3) + x/y^2x(x+3) + x^2y(x+3)/y^2x(x+3)$

-combine into one fraction

$= 2y^3+x-x^3y-3x^2y/xy^2(x+3)$algebraic divisiondegree - the highest power of x in the polynomial (e. g. the degree of $4x^5+6x^2-3x-1$ is 5)

divisor - this is the thing you're dividing by (e. g. if you divide x^2+4x-3 by $x+2$, the divisor is $x+2$)

quotient - the but you get when you divide by the divisor (not including the remainder)$f(x)= q(x)d(x)+r(x)$ - way of writing a polynomial$q(x)=$ quotient

$d(x)=$ divisor

$r(x)=$ remainderpartial fractions= opposite of adding fractions

if the fraction is over two different brackets then the partial fraction will be in the form A/bracket 1 +B/bracket 2

if the fraction is over three different brackets then the partial fraction will be in the form A/brackt1 +B/bracket2 +C/bracket3

if the fraction is over one bracket squared and another bracket then the partial fraction will be in the form A/bracket 1 squared + B/bracket 1 +C/bracket 2difference of two squares denominatorse. g. $21x-2/9x^2-4 = 21x-2/(3x-2)(3x+2) = A/(3x-2) + B/(x+2)$parametric equationssplit up x and y into separate equations cartesian equation = single equation linking x and y

parametric = x and y are defined separately with a third variable called a parameter usually t or thetause parametric equations to find where graphs intersectintersect the x axis , y= 0

intersect the y axis , x= 0

intersect the line ay= bx +c , plug the parametric equations into the values of y and x and then rearrange and find the values of the parameter which you then plug into the parametric equations to find the corresponding x and y valuesconverting parametric equations into cartesian equations`1- rearrange one of the equations to make the parameter the subject, then substitute the result into the other equation

or

2-if your equations involve trig functions , use trig identities to eliminate the parameterexamples1- e. g. x= t+1 y= t^2-1

t= x-1

y=(x-1)^2-1

y= x^2-2x

2- e. g. x= 1+sin(x) , y= 1-cos(2x)

use trig function cos(2x)= 1-2sin^2(x)

y= 1-cos(2x)= 1-(1-2sin2(x))= 2sin^2(X)

sin(x)= x-1 so y= 2sin^2(x)

y= 2(x-1)^2 = 2x^2-4x+2

binomial expansionformula in the book

used for when the power is a fraction or is negative

if the constant isn't 1 then you need to factorise it first (e. g. (3-x)^4 will become (3(1-x/3))^4 which becomes 81(1-1/3x)^4validityif the power is positive , the expansion is valid for all values of x

if the power is negative or a fraction, the expansion of (p+qx)^n is only valid when | qx/p|<1e. g. 1(1+x)^-1 is valid or when | x|<1e. g. 2 (1+2x)^1/3 is valid if | 2x|<1 = 2| x|<1 = | x|<1/2dy/dx of sin(x)= cos(x)dy/dx of cos(x)= -sin(x)dy/dx of tan(x)= sec^2(x)rule for dy/dx of trig functionsonly work in radiansuse the chain rule with sin/cos/tane. g1 differentiate y= cos(2x) + sin(x+1)

y= cos(2x) becomes y= cos(u) , u= 2x

dy/du= -sin(u) du/dx= 2 so dy/dx = -2sin(2x)

y= sin(x+1) becomes y= sin(u), u= x+1

dy/du = cos(u) du/dx= 1 so dy/dx = cos(x+1)

therefore dy/dx = cos(x+1)-2sin(2x)

e. g. 2 find dy/dx when x= tan(3y)

x= tan(u), u= 3y so dx/du = sec^2(u) du/dy = 3

dx/dy = 3sec^2(3y) so dy/dx = 1/3sec^2(3x) = 1/3cos^2(3y)

dy/dx of inverse reciprocal functionscome from the quotient rule differentiation of 1/sin(x), 1/cos(x) and 1/tan(x)dy/dx of cosec(x)=-cosec(x)cot(x)dy/dx of sec(x)= sec(x)tan(x)dy/dx of cot(x)=-cosec^2(x)use the chain, product and quotient rules with cosec, sec and cote. g. 1 find dy/dx of y= sec(2x^2)

= CHAIN RULE as it is a product of a product

so y= sec(u) u= 2x^2

dy/du= sec(u)tan(u) du/dx = 4x

therefore dy/dx = 4xsec(2x^2)tan(2x^2)

e. g. 2 find dy/dx of y= e^x(cot(x))

= PRODUCT RULE as it is a product of two functions

u= e^x v= cot(x)

du/dx = e^x dv/dx =-cosec^2(x)

dy/dx = u(dv/dx)+v(du/dx)

= e^x(cot(x)-cosec^2(x))

differentiating parametric equationsdy/dx= dy/dt / dx/dt

can use this to find the gradient of tangents and normal (neg reciprocal of tangent)implicit differentiationimplicit relation= an equation thats in the form f(x, y)= g(x, y) rather then y= f(x)steps for implicit differentiation1. differentiate the x^n values with respect to x as usual

2. differentiation the y^n values by d/dy so the result is d/dy of f(y) multiplies by dy/dx

3. use the product rule to do ones with x and y

4. rearrange the equation to make dy/dx the subject

can use this to find the gradient

intergration of sin(x)= -cos)x + cintegration of cos(x)= sin(x) + chow to integrate trig functionsfollow the rule given

if the x of the starting equation's x has a coefficient that isn't 1, you just divide by the coeffiecnt when you integratesome fractions integrate to lnS f^-1(x)/f(x) dx = ln| f(x)| + cthis works for some trig functions tootan(x) =

sin(x)/cos(x)

dy/dx (cos(x)) =-sin(x) therefore S sin(x)/cos(x) = -ln| cos(x)| +c

S cot(x) dx = ln | sin(x)| +c

S cosec(x) dx = -ln | cosec(x) + cot(x)| +c

S sec(x) dx = ln | sec(x) + tan(x)|

chain rule in reversefind S 6x^5(e^x^6) dx

if you differentiate the bracket you would get 6x^5(e^x^6) therefore as only differentiating the bracket gets this answer so the answer is the bracket = e^x^6 +cintegration by substitutionon products of two functions

1. find du/dx and rewrite it so dx is on its on

2. rewrite the original integral in terms of u and du

3. integrate as normal

for definite integrals you have to change the limitsintegration by partsS u(dv/dx) dx = uv -S v(du/dx)integrate ln(x)rewrite at 1xln(x)

u= ln(x) dv/dx= 1

du/dx= 1/x v= x

therefore S ln(x) = xln(x) -Sx(1/x) dx = xln(x) -S 1 dx = xln(x) -x +cintegrating partial fractionssplit into partial fractions

when you integrate each fractions they are all going to be Aln(bracket1) +Bln(bracket2)... +c

which you can rewrite as ln((bracket1)^A(bracket2)^B) +cdifferential equations- solving by integrationwrite the differential equation in the form dy/dx = f(x)g(y)

rewrite the equation in the form 1/g(y) dy = f(x) dx

integrate both sides

rearrange into a nice form and remember to +cstarting conditions occur whent= 0decreasing =-k (rate of decrease is proportional to number of...)vectorshave a size and a direction

scalars are just quantities

length of the vector is the magnitudea, 2a, 3aparallel as they are just multiplied by a scalarposition vectorswhere the point liesa unit vector isany vector with a magnitude of 1 uniti and j unitsi is the x axis

j is the y axis

(k is the z axis)column vectorsway of writing a vectorpythagoras' theoremfind vector magnitudes

the magnitude of a vector is written | a| the distance of a point from the originsquare root of (x^2 + y^2 + z^2)distance between pointssquare root of (x1-x2)^2 + (y1-y2)^2 + (z1-z2)^2equation of a line through a point and parallel to another vectora straight line which goes through point A and is parallel to vector B has the equation

r= A+tB

where r= position vector of a point on the line

A= position vector of point Aequation of a line passing through two pointsr= c+t(d-c)point of intersection of two linesif two lines intersect then there will be a value of t for each equation which makes the same r

you know if they are parallel if they are multiples of each other

if there is no point of intersection then they are skewscalar producta. b = | a|| b| cos(x)

used to calculate the angle between two lines

rearranges to cos(x)= a. b/| a|| b|

two lines that are perpendicular , the angle will be 90 therefore cos(90) = 0 if they are parallel it would be cos(0) so a. b = | a|| b| a.

ba1b1+a2b2+a3b3using the scalar product to find the angle between two vectorse. g. find the angle between the vectors -i-6j and 4i+2j+8k

a=-i-6j b= 4i+2j+8k

scalar product of the vectors

a. b = (-1x4) + (-6x2) + (0x8) = -4-12+0=-16

magnitude = | a| = root 37 | b| = root 84

cos(x)=-16/root 37 x root84 so x= 106. 7 degresif you are finding the angle between two linesyou would use the b bit from r= a+tbproving lines are perpendicularscalar product = 0 ON