The applications of arrow debreu model economics essay

According to Elroy Dimson and Massoud Mussavian (1999), Arrow-Debreu model was developed as a model of general equilibrium that has been fundamental to economics and finance. Compared to earlier models, the Arrow-Debreu model basically generalized the notion of a commodity, differentiating commodities by time and place of delivery. For example, “ apples in Malaysia in July” and “ apples in Singapore in June” are considered as different commodities.

Kenneth J. Arrow (1951) and Gerard Debreu (1951) work together to produce the first rigorous proof of the existence of a market clearing equilibrium, given certain restrictive assumptions. This field of research has had a profound impact not only on economic science, but also on financial markets, institutions and businesses all over the world. It often used as a general reference for other microeconomic models. As Ramu Gopalan (2008) stated, the pioneering work of Arrow and Debreu has had an enduring effect on the study of financial aspects of the economy in a general equilibrium framework. One of their key contributions is to introduce time and uncertainty into general equilibrium models. The Arrow-Debreu model was established since 1950s, many researchers had extended this model to both economics and financial economics. Although this model is criticized by various eminent economists, the dedication of this model in the history is indestructible.

In this assignment, we are going to discuss the applications of Arrow-Debreu model majoring in the financial economics. The purpose of this assignment is to find out and understand more about the contributions of this model to financial theory. The applications of Arrow-Debreu model will be listed out

and discussed further. Journals will be shown and summarized out in order to support our discussion. Finally, the last section in this assignment is the conclusion.

2. 0 Background studies

In this assignment, the applications of Arrow-Debreu model in financial economics will be discussed. But before that we have to know what the Arrow-Debreu model is.

2. 1 Arrow-Debreu Model

Arrow-Debreu model, also referred as Arrow-Debreu-McKenzie model (ADM model), is the fundamental model used in the General (Economic) Equilibrium Theory. It is named after its originator who are Kenneth J. Arrow (b. 1921) and Gerard Debreu (1921-2004) on " Existence of an Equilibrium for a Competitive Economy" as well as Lionel W. McKenzie (b. 1919). As what stated in the Farlex Financial Dictionary (2009), it says that this model is one of the most general models of competitive economy and is a crucial part of general equilibrium theory, as it can be used to prove the existence of general equilibrium (or Walrasian equilibrium) of an economy. Once we can prove the existence of such an equilibrium, it is possible to show that it is unique under certain conditions, but not in general. Furthermore, Arrow went on to extend the model to deal with the issues relating to uncertainty, stability of the equilibrium, and whether a competitive equilibrium is efficient.

2. 2 Applications of Arrow-Debreu Model

Arrow-Debreu model leads to a huge impact on economics and financial economics. First of all, it solves the long-standing problem of proving the existence of equilibrium in a Walrasian (competitive) system.

This model analyzes the exact situations of those markets that are very competitive. In economics, Arrow-Debreu model suggests that a set of prices such as aggregate supplies will equal to aggregate demands for every commodity under certain assumptions made about the economic conditions (i. e. perfect competition and demand independence).

Formulated in a purely mathematical form, the Arrow-Debreu model can be easily modified into spatial or intertemporal models with proper definition of the commodities based on the commodity’s location or time of delivery. When commodities are specified to be conditional on various states of the world, the Arrow-Debreu model can easily combine expectation and uncertainty into the analysis. Besides, theoretical extensions and applications have been made to analyze financial and monetary markets and international trade, as well as other subjects. With a general equilibrium structure, the model is applicable in evaluating the overall impact on resource allocation of policy changes in areas such as taxation, tariff, and price control. Moreover, it applies to all general equilibrium models that are heavily dependent upon accurate mathematical proofs.

In the field of financial economics, Arrow Debreu represents a certain kind of securities product which named as Arrow-Debreu security. This distinguished concept is a good teaching tool to understand the pricing and hedging issues

in derivatives analysis. On the other hand, the Arrow-Debreu Model is also used in areas like financial engineering. But it has turned out to be very limited, especially in the multi-period or continuous markets.

The model has been subject to the criticism that many of the assumptions it makes do not fit the workings of the real economy. However, the truth is that the Arrow-Debreu Model is very important for the derivative industry and helps the industry to grow at a rapid pace.

3. 0 Literature Review

In previous section, we have mentioned some applications of the Arrow-Debreu model both in the field of economics and financial economics. Now, the applications of this model majoring in financial economics will be discussed further.

The functions of Arrow-Debreu model can be divided into six categories, asset pricing model, equity risk premium, corporate finance, Modigliani and Miller Theorem, Arrow-Debreu security and others.

3. 1 Asset-pricing model

From the studies, most of the Arrow-Debreu model’s applications are commonly used in shaping the asset-pricing model. Arrow-Debreu model was acted as an origin which gives the insight that consumption in different future states could simply view as different consumption goods according to Elroy and Massoud (1999). This result is proved and can be seen through various researchers’ journals. It is undeniable that the Arrow-Debreu model

plays an important role in constructing the asset-pricing model. The evidences are given in following paragraphs.

Based on the journal of " Asset Pricing at Millennium" written by John Y. Campbell (2000), he stated that theoretical and empirical developments in asset-pricing has taken place within a well establish paradigm for the last twenty years. While the well establish paradigm that he mentioned here is referred to the Arrow-Debreu model. Same as Franklin Allen (2001), he indicated that asset-pricing models are typically special cases of neoclassical Arrow-Debreu model. In the traditional Arrow-Debreu model of resource allocation, firms and households interact through markets and financial intermediaries play no role. On the other hand, the key element of the analysis in the modern version is the stochastic discount factor, which incorporates the Arrow-Debreu state prices and allows the assets to be priced. He also commented that this approach and the focus on the risk-return trade-off have allowed a rich interplay between the empirical and theoretical work. The equity premium puzzle is given as an example of special cases within the Arrow-Debreu framework in order to support his statement.

Moreover, Elroy and Massoud (1999) narrated the historical development of asset pricing and derivative valuation on " Three Centuries of Asst Pricing". He pointed out the success of conceptual framework that setting up the theory of asset pricing is down to Arrow (1953)'s hard work. Dissatisfied with the current Arrow-Debreu framework, Arrow built up a series of contingent claims that follow the resolution of uncertainty to explain how one can achieve markets that are almost complete.

Varian (1985) analyzed the impact of divergence of opinion on asset prices in an Arrow-Debreu economy. By considering the Arrow-Debreu model with agents who have different subjective probabilities, he compares and concludes the differences of opinion in an Arrow-Debreu contingent claim context. Based on his journal " Divergence of Opinion in Complete Markets: A Note", three results were established. He concluded that in practice, increased dispersion of beliefs will generally be associated with the reduced asset prices in a given Arrow-Debreu equilibrium. Also, he uses this model to show that other things equal, if risk aversion does not decrease too rapidly, then assets with more dispersed opinion will have lower prices or vice versa.

P. Bossaerts and C. Plott (2004) had done six financial markets experiments of testing two of the most basic propositions of modern asset pricing theory. The Arrow-Debreu model and the Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM), these two theoretical models are used to be the framework of their experiments. In the end of their experiments, they discovered a swift convergence towards equilibrium prices of Arrow-Debreu model or the CAPM. This discovery is significant because they use the subjects that lacked of information to intentionally set the asset prices. Sometimes, the equilibrium is not found to be robust which clearly shows a result of deviations of subjective beliefs from objective probabilities. However, they still find the evidences that prove this does not destroy the tendency for markets to equilibrate as predicted by the theory.

3. 2 Equity Risk Premium

Next, the Arrow-Debreu model is applied to explain the equity risk premium.

In an attempt to explain the equity risk premium, Rajnish Mehra and Edward

C. Prescott (1985) developed an Arrow-Debreu asset pricing model. They found that historically the average return on equity has far exceeded the average return on short-term debt and Treasury bills. Thus, they try to use the Arrow-Debreu model to interpret this situation. In the end of the journal " The Equity Premium: A Puzzle", they concluded that only those equilibrium model with friction (i. e. non-Arrow-Debreu models) will be the one that successfully explain both high equity risk premium and low risk-free returns.

However, Rietz (1988) overthrew the conclusion of Mehra and Prescott (1985) in " The Equity Risk Premium: A Puzzle". He mentioned that the reason for them to reject the Arrow-Debreu model is their specifications which cannot explain the high equity risk premium and low risk free returns that characterize the U. S. economy. Hence, he re-specified their model to include a low-probability, depression from a high return of compensation for the extreme losses during the market crashes, captured those possible effects from the market crashes and finally successfully proved that these crashes allow it to explain both high equity risk premium and low risk free returns without abounding the Arrow-Debreu paradigm as well as not altering their model's attractive features. In the journal " The Equity Risk Premium: A Solution", he explained further that it does so with reasonable degrees of time preference and risk aversion provided the crashes are apparently severe and not too unimaginable.

3. 3 Corporate Finance

According to Jean Tirole (2006), he specified that a substantial and important body of empirical work has provided a clearer picture of patterns of

corporate financing and governance, and of their impact for firm behaviour and macroeconomic activity. One of them is the Arrow-Debreu model.

During 1970s, the dominant Arrow-Debreu model of frictionless markets (presumed perfectly competitive and complete, unhampered by taxes, transaction costs, as well as informational irregularity) can prove to be a powerful tool for analyzing the pricing of claims in financial markets, but little about the firms' financial choices and about their governance. Besides, in the complete market paradigm of Arrow (1951) and Debreu (1951), the financial claims' returns depend on some choices such as investments, are assumed to be contractible and therefore are not affected by moral hazard. In Jean (2006)'s opinion, financial markets were not plagued by problems of asymmetric information because investors agree on the distribution of a claim's returns. Viewed through the Arrow-Debreu lens, he identified that the key issues for financial economists are the allocation of risk among investors and the pricing of redundant claims by arbitrage.

Michael J. Brennan (1995) also clarified that the abstract simplicity of the Arrow-Debreu model yields few insights for corporate finance beyond the value additivity principle that was used to refute the conventional wisdom that conglomerate mergers will add value to the company through the corporate diversification.

3. 4 Modigliani and Miller Theorem (M-M Theorem)

Another application of Arrow-Debreu model is related to M-M theorem which devised by Franco Modigliani and Merton Miller (1958). This theorem explains that a firm's financial structure is irrelevant under certain

conditions, Arrow-Debreu environment. The value of a financial claim or a firm which equal to the sum of the values of the claims it issues is thus equal to the value of the random return of this claim or the firm computed at the Arrow-Debreu prices (the prices of state-contingent securities). Therefore, Arrow-Debreu model is used as a fundamental of economy in formulating M-M theorem.

3. 5 Arrow-Debreu Security (State Contingent Claim)

Mostly, Arrow-Debreu security will be the answer of the applications of this model majoring in financial economics if we searching it through the internet. Based on the journal " A re-examination of the Modigliani-Miller theorem" written by Joseph E. Stiglitz (1969), in a section entitled " Arrow-Debreu securities", he not only showed the M-M theorem in a complete markets setting but also mentioned about the Arrow-Debreu model under uncertainty in which individual can buy or sell the promises to pay if a given state of the world occurs. This shows a direct relationship between the Arrow-Debreu model and the Arrow-Debreu security. Through the calculation, he observed that if he takes literally the Arrow-Debreu definition of a state of nature, there is undoubtedly will be more states of nature than firms and most of these states are similar with each other. An example, " variation in the return on stocks can be explained by the business cycle", is given to support his statement.

Robert E. Lucas (1984) analyzed the unified theories of money and finance on " Money in the Theory of Finance". He examined and commented that financial and monetary theory have different objective, however, the desirable theoretical " unity" may be, one can identify strong forces that will

continue to pull apart these two bodies of theory. He mentioned that the theory of finance is conducted almost entirely within the Arrow-Debreu contingent claim framework such as the three pillars of modern financial theory which have been reformulated in contingent claim terms. Besides, he wrote that the applications of the Arrow-Debreu contingent claim formulation of a competitive equilibrium for an economy operating through time is subjected to stochastic shocks. In the end, he concluded that the power in applications of the contingent claim point of view is obviously evident in finance, will be as usefully applied to monetary theory. One more thing that he suggested is the source of this power which is the ability of this framework to permit the reduction of the study of asset demands to the study of demands for the more fundamental attributes to which assets are claims.

3. 6 Others

Apart from those above categories, Arrow-Debreu Models can be used for other purpose. For instance, it acts as a fundamental to explain the pattern of trade, to formulate the fixed price equilibrium or to find out whether the financial markets are arrangements for risk-sharing. Furthermore, it is extended further to analyze the restrictions and developed further that include a sequential market model with the financial markets.

One of the section in the journal “ Differences of opinion in financial markets” written by Hal R. Varian (1989), an Arrow-Debreu contingent consumption model of the sort studied by Milgrom and Stokey (1982) was examined. Through the mathematical calculation and the analysis of the consequences for assets market equilibrium based on the Arrow-Debreu

model, it ends with a similar result, prices are determined by information, but the pattern of trade is determined by differences in opinion. In order to establish the important difference for trade is the opinion, he analyzed some of its consequences for assets market equilibrium. At last, he stressed that the volume of trade in an Arrow-Debreu model is due primarily to the differences of opinion.

Next is the Claus Weddepohl (1983). He discussed and addressed the development of the theory of general equilibrium during the last twenty-five years. Considering and analyzing the Arrow-Debreu model with futures markets, he showed the result that this model gives rise to temporary equilibrium models. He stated that the fixed price equilibrium models are formulated through the study of these models and the study of the stability of price adjustments. The simple fixed price equilibrium model as defined by Barro and Grossman (1971) and Malinvaud (1977) is what he emphasized in the journal “ Developments in the Theory of General Equilibrium”.

Ouattara (1994) applied the Arrow-Debreu model to the small villages in the McCarthy Island Division South (MID-South) of The Gambia to find out whether financial markets are arrangements for risk-sharing. The main objective of risk-sharing is to verify that observed consumption patterns are consistent with patterns predicted by insurance models. The Arrow-Debreu full insurance model focuses on consumption smoothing across different states of nature at each particular point in time through state-contingent contracts. In the end of his research, the results supported the hypothesis that state-contingent loans are accepted in rural Gambia and there is full risk-sharing among participants in the financial markets.

Peter H. Friesen (1979) extended the Arrow-Debreu model to financial markets which include the sequential market model. It is done by dropping the contingent contracts from the Arrow Debreu model, leaving only a sequence of spot markets for commodities. This leads to an inefficient market structure but efficient for sequence of stock markets and option markets. The purpose of the journal " The Arrow-Debreu Model Extended to Financial Markets" is to develop further the Arrow-Debreu model. The method that he used is through the extension of Arrow (an equilibrium in one model was constructed from that of another). For example, financial securities, Arrow certificates can be constructed from options on common stock and the advantage in the general equilibrium theory of financial markets, are the proofs of using Arrow's method.

On the other hand, the sequential market model for which equilibrium are constructed from the Arrow-Debreu equilibrium derived in Debreu (1951). From the fact that Arrow-Debreu equilibrium exist, it follows by construction that equilibrium for this model exists. It also follows that the equilibrium is efficient. Peter (1979) stressed that such models are used both to study financial markets and to explore the effects of a gradual resolution of uncertainty. Lastly, he concluded that it not only shows the close relationship between these two models but also reminds us that the potential value of finite-horizon Arrow-Debreu models for the study of sequential economies.

Investors in financial markets face several restrictions apart from wealth constraints. So, we have the right to understand the restrictions in a general competitive equilibrium. Based on the journal " Contributions to Intertemporal Models in Financial Economics" written by Ramu Gopala

(2008), the Arrow-Debreu model was extended further for the usage of analyzing those restrictions.

He indicated that the traditional Arrow-Debreu model can be extended to a more realistic setting. Following Angeloni and Cornet (2006), this extension of the Arrow-Debreu model in the multi-period setting with restricted participation is established. Arrow-Debreu model was used to elaborate, to compare, to extend and to emerge for shaping those important finance theories.

4. 0 Supporting Theories

In this section, the theories that are related to Arrow-Debreu model will be further discussed. Apart from that, in order to let us have a more complete picture about this model, the theories that we displayed previously will also be stated and explained, as well as deliberated further.

4. 1 General Equilibrium Theory

General equilibrium theory is the core of economic theory. Before the Arrow-Debreu model is established, this theory has been proposed by L. Walras (1874). As reported by Arrow and Debreu (1954), he was the one who first formulated the state of economic system at any point of time as the solution of a system of simultaneous equations representing the demand for goods by consumers, the supply of goods by producers, and the equilibrium condition that supply equal demand on every market. In other words, Walras (1874) is the pioneer who first attempts to model the price for a whole economy.

Walras uses mathematics to construct a complete structure of general equilibrium theory. This research has lead to results in contribution to neoclassical economics. However, the mathematics that he used to set up the foundation of this theory was unstable due to the existence of general equilibrium does not solved in a satisfactory manner. Hence, theoretically, if he cannot prove this existence, then this theoretical system will become meaningless. General equilibrium theory is therefore developed and improved by Vilfredo Pareto (1897), John R. Hicks (1939), John Von Neumann (1937), Paul A. Samuelson (1941), Kenneth J. Arrow (1954), Gerard Debreu (1954), Lionel W. McKenzie (1954) and others, which becomes an integral part of economics.

4. 2 Fundamental Theorems of Welfare Economics

Before the Arrow and Debreu began their famous collaboration, both of them had proved the same theorem which is the Fundamental Theorems of Welfare Economics or First and Second Welfare Theorems. There are two fundamental theorems of welfare economics. The first theorem states that every general equilibrium involves a Pareto efficient allocation of resources under the three assumptions. The three assumptions are if there are no externalities, all agents are price-taker, and prices for each good are known to each agent. While the Pareto efficient named after Vilfredo Pareto (1897), is a type of efficiency that results if one person cannot be made better off without making someone else worse off.

The First Welfare Theorem is viewed by many economists as the formalization of Smith’s Invisible Hand. As Makowski and Ostroy (1995) stated, it provided a set of sufficient conditions for a price system to

efficiently coordinate the economic activity. Besides, this theorem supports the case for non-intervention in ideal conditions. For instance, the outcome is said to be Pareto efficient if we let the markets to do the work.

The Second Welfare Theorem says that if preferences are well-behaved (especially convex) then every Pareto efficient allocation can be supported by a general equilibrium set of prices, given a suitable reallocation of the endowment. Referring to Varian (1985), this theorem effectively said that if you think an equilibrium is unfair, just move the endowment of the economy and a different general equilibrium will be obtained. Due to the convexity, the second theorem is stronger than the first theorem. The difference between these two theorems is the second theorem requires existence of general equilibrium from all endowment points, whereas the first theorem required only that if a general equilibrium existed it was efficient.

According to Michael A. S. Guth (1994), Arrow (1951) provided a rigorous proof of the connection between competitive equilibrium and Pareto optimal. Gerard Debreu (1951) introduced convex analysis methods into welfare theory and independently proved the same theorems. As a result, these theorems have an important relation to Arrow-Debreu model; the existence of solutions to a competitive equilibrium is finally solved.

4. 3 Theories in relation to the Restriction of General Competitive Equilibrium

Previously, the Arrow-Debreu model was used to analyze the restrictions. In order to understand those restrictions in the general competitive equilibrium framework, tracing back those following theories is necessary for us to know

how the Arrow-Debreu model was used to be compared and proved other economic theory.

4. 3. 1 Radner Equilibrium

Roy Radner said that the Arrow-Debreu model is not originally put forward for the case of uncertainty, but a powerful device introduced by Arrow (1953), and further elaborated by Debreu (1953), enabled the theory to be reinterpreted to cover the case of uncertainty about the availability of resources and about consumption and production possibilities. Hence, he extends the Arrow-Debreu equilibrium and forms an economic concept-Radner Equilibrium.

Radner (1972) is the first who considers the general equilibrium with incomplete markets. He shows that unlike the Arrow-Debreu models, the possibility of trading commodity futures for every contingency is sufficient to enable income transfers across all spots. In addition, the assumption that he made, short-sales of these contracts are limited for every agent, is a driving force in his proof of the existence of a general financial equilibrium. This can be seen as the first attempt to incorporate this idea in their asset market participation.

4. 3. 2 Concept of Constrained Pareto Optimality

Radner Equilibrium, however, is imperfect. Oliver D. Hart (1975) uses some disturbing but perceptive counter-examples to display some of the weaknesses of Radner’s concept of equilibrium. He showed that existence of such an equilibrium cannot be proved under the standard Arrow-Debreu assumptions. He specified that when the asset returns are price dependent,

the market sub-space may not be continuous in the spot prices which may lead to discontinuous demand functions. This reason causes a failure of the existence of Radner equilibrium. In other words, an equilibrium may not be Pareto efficient in the case of incomplete markets which shapes the concept of Constrained Pareto Optimality.

4. 4 Limitations of Arrow-Debreu Model

Although the Arrow-Debreu model has many influences on either economics or finance, however, there are the limitations. There are three limitations of Arrow-Debreu model.

In this model, it excludes the trade in shares of firms because the stock certificate is not an Arrow-Debreu commodity. When the descriptions are so precise that further refinements cannot yield imaginable allocations which increase the satisfaction of the agents in the economy, then the commodities are called Arrow-Debreu commodity. Trading in shares of firms cannot be classified as Arrow-Debreu commodity due to its possession entitles the owner to additional commodity which he need not obtain through exchange.

Bankruptcy is not allowed in Arrow-Debreu equilibrium. All agents must meet their budget constraints. In a game theoretic formulation of equilibrium, it is achieved by enforcing an infinite bankruptcy penalty. Since every Arrow-Debreu equilibrium is Pareto efficient, there would be no benefit in reducing the bankruptcy penalty to the point where someone might choose to go bankrupt.

Money does not appear in this model. Although the reasons for the existence of money in real life are already taken care of in the Arrow-Debreu model,

money does not affect the allocations of commodities. Therefore, there is no point in making the role of money explicit in the Arrow-Debreu model.

5. 0 Conclusion

In fact, Arrow-Debreu model is not simultaneously created by K. J. Arrow and Gerard Debreu. Debreu is the one who extends further the Arrow's pure exchange model in several important ways. Their contribution in formulating Arrow-Debreu model has laid a foundation for economic theory.

The application of Arrow-Debreu model emphasizes more on the general competitive equilibrium framework of the economics. From the studies, we notice that most of its application majoring in the financial economics is act as a fundamental theory or economy in shaping the asset pricing model. Other functions like analyzing the market structure, risk and etc, also show that the usefulness of this model.

Besides, it is applicable in evaluating the impact of all uncertainties with a general equilibrium structure. The analyzers use a series of mathematical equation to prove their statements. The pioneering contributions of Arrow and Debreu have forever changed the way economic theorists formulate uncertainty models. After more than forty years of analysis and extensions, their general equilibrium framework and approach continues to be the starting point for new theories on the operation of competitive markets under uncertainty.

As a conclusion, it is undeniable that the Arrow-Debreu model had turn on a new leaf in the history of economics. It is the modern concept of general equilibrium in economics which indirectly set up several important theories.

Nothing is perfect in this world. Of course, Arrow-Debreu model as well. Some assumptions have to be made. Criticism also may happen. However, those brilliant economists or researchers still can use this model to formulate their own theory and then ends up with a perfect ultimate theory in both economics and finance. Lastly, mistakes, problems, and weaknesses should be pointed out, corrected and improved so that Arrow-Debreu model can be applied and developed effectively.