

The also adheres to
poisson's postulates,
talked about



**ASSIGN
BUSTER**

The Poisson distribution as an approximation of the binomial

distributionIntroductionWhen I was reading up on radioactive decay, I found out that the probability of decay could be expressed in terms of a discrete probability distribution. I had to look it up further because I had just recently learned about probability distributions in math. What I found out was intriguing; the number of atoms decaying in a specific interval of time is actually a discrete variable. It also adheres to Poisson's postulates, talked about later, so it can be expressed in terms of Poisson distribution.

However, when what I noticed was that that some research papers and online resources would use Poisson distribution, and some would use the binomial distribution. This intrigued me because if one could be used over another, then that means the two should be synonymous in nature. This goes against my previous knowledge on the two because I thought that since they both have different formulas, they should be two very different concepts. After consultation with my physics teacher, I was told that probability distributions would come up in higher education engineering courses.

Being an aspiring engineer, I had to explore the differences between binomial and Poisson distribution and understand why they can be used side by side. Poisson distributionThe Poisson distribution was first introduced in the 18th century by mathematician Siméon Denis Poisson as a way to research on the number of wrong court decisions over a period of time. In a more general sense, this distribution tells us the probability of the sum of successful independent Bernoulli trials given a fixed interval of time.

A Bernoulli trial is a statistical experiment where there are only two possible outcomes, either success or failure. Graphically, it provides a discrete probability distribution function where each point in the y-axis gives the numerical probability of a discrete random variable X . It is typically denoted as: Discrete probability

The Poisson distribution works with discrete variables, so it is worth discussing what it is first. Discrete variables:

- Have a finite set of data
- Are obtained by counting (and is countable)
- Are non-mutually exclusive
- Have a complete range of numbers
- Are represented by distinct, isolated points in a graph

One of the simplest statistical examples of the discrete variable is the probability of getting number of heads when a coin is tossed times. Suppose we have a fair coin, the probability of getting number of heads when the coin is tossed 2 times is: Explanation

Probability (fraction)	Probability (decimal)
0T	0.25
1HT, TH	0.5
2H	0.25

Because of the nature of the discrete variable, its probability distribution can often just be expressed in a tabular form. Poisson's postulates

There are several assumptions to be made when using the Poisson distribution. When these assumptions are met, then the Poisson discrete probability distribution can be used.

- 1.

2. The probability of success is the same throughout the whole experiment
3. The probabilities are independent of one another
4. The probability of a success happening over a small time period is essentially the value of that time period
5. The probability of more than one success in a small time period is essentially 0
6. The rate of success is only dependent on the length of the interval of time

The experiment at hand is a part of a Bernoulli

trial One thing worth adding when comparing the probabilities of a random

<https://assignbuster.com/the-also-adheres-to-poissons-postulates-talked-about/>

variable in real life and in calculations is that if there is a large discrepancy between the two sets of data, there must be an external factor coming into play when the statistical experiment was done. Poisson distribution as a derivation from binomial distribution The binomial distribution has a more specific definition, which is the probability of having successful outcomes out of Bernoulli trials.

For the probability that a discrete random variable happens times, it is denoted as: $P(X = k)$. What I found out after my exploration amazed me; the Poisson distribution is actually just a derivation from the binomial theorem for when n is large and p is small. It is a mathematical limit of the binomial distribution. The Poisson distribution thus has to be derived from: To come up with a correct derivation, the calculations must adhere to Poisson's postulates. Because the probability of success is identical throughout the whole experiment, this implies that in n number of trials, the expected value is np , which is also the definition of mean (μ). Rearranging it: Substituting μ into the equation: Now the first two terms can be taken out and manipulated.

It can be rewritten as: Because both the numerator and denominator now have n they can be cancelled out, leaving the following: From here, it can be noted that both the numerator and denominator have n number of terms. However, as n reaches infinity, the value of this whole fraction approaches 1. Therefore, it can be said that the value of the first two terms is 1. The last term can be divided into two parts: For the first part, an expression for Euler's number can be used: For the second part, since n is in the denominator and it is approaching infinity, the value of the fraction will approach

1. Putting them all together, including the constants taken out earlier: This <https://assignbuster.com/the-also-adheres-to-poissons-postulates-talked-about/>

simplifies to: Hence the Poisson distribution is denoted as: In summary, the Poisson distribution is a condition of the binomial theorem where the number of trials approaches infinity and the probability of success approaches 0. Euler's number in probability What I find very interesting is the fact that Euler's number suddenly popped out when deriving for the Poisson distribution.

It is clearly one of the most fascinating, important, and fundamental constants in mathematics. Unsurprisingly, it has its applications in probability theory and it relates directly to the Poisson distribution. For a large n , the probability of getting no successful outcomes is approximately.

This expression can actually be proven to be correct by inputting the parameters in both the Poisson and binomial distributions. Let us take a look at how that could work with an example: A student is playing a random number generator, which has a range of numbers from 1 to 100; the teacher says that if after 100 tries the number 65 does not appear, the student can go home. What is the probability that the student can go home? We have to first find out the probability of success, which in this case is the number 65 appearing. The probability of it happening is 1 in 100. So now, What we want to find is the probability when there are no successful outcomes, so. This is the same value of the limit shown above. Even in other cases wherein n , the probability should still be.

This is because as n and p , the value for e^{-np} will be e^{-np} . We can also try this with the binomial theorem, where there are no limiting assumptions made. Which is essentially: However, the actual probability is e^{-np} , which is a very close

approximation to Euler's number. For even larger values of, the probability should get close and closer to. This result of this example justifies the use of the Poisson distribution as the equation used in substitution to the binomial theorem when reaches a very large number.

It also signifies just how important Euler's number is in the limit theorem, and the credibility of its usage in the Poisson distribution. Approximation of the Poisson distribution to the Binomial distribution I also graphed these distributions so that it can be visualized. However, because I did it in my laptop's default graphing software, there were some things that I had to change. I was not able to change the y and x axes' variables.

Due to this, I had to change some variables so that the equations would be able to be plotted. Because the probability is the independent variable, P was changed to y. The dependent variable in the equations is the number of successful Bernoulli trials, so was changed to x. For the example above:
Binomial: red Poisson: blue Poisson: For the binomial theorem, I had to expand the first part of the equation because the software did not recognize it.
Binomial: As seen from the graph above, the graphs are almost identical. However at closer inspection: The x-intercepts are the values that were obtained earlier It can now be clearly seen that there is still some discrepancy between the two. Mean, variance, and mode