Cfa- economics

Economics

ADS It has two variables, share price S and time t. However, there is a second derivative only withrespectto the share price and only a first derivative with respect to time. Infinance, these type equations have been around since the early seventies, thanks to Fischer Black and Myron Schools. However, equations of this form are very common in physics Physicists refer to them as heat or diffusion equations. These equations have been known In physics for almost two centuries and, naturally. Scientists have learnt a great deal about them.

Among numerous applications of these equations in natural sciences, the classic examples are the models of Diffusion of one material within another, like smoke particles in air, or water pollutions; Flow of heat from one part of an object to another. This is about as much I wanted to go into physics of the BBS equation. Now let us concentrate on finance. What Is The Boundary Condition? As I have already mentioned, the BBS equation does not say which financial instrument it describes. Therefore, the equation alone is not sufficient for valuing derivatives.

There must be some additional information provided. This additional information is called the boundary conditions. Boundary conditions determine initial or final values of some financial product that evolves over time according to the PDP. Usually, they represent some contractual clauses of various derivative securities. Depending on the product and the problem at hand, boundary conditions would change. When we are dealing with derivative contracts, which have a termination date, the most natural boundary conditions are terminal values of the contracts.

For example, the boundary condition for a European call Is the payoff function V(SST, T) = Max(SST-DE) at expiration. In financial problems, it is also usual to specify the behavior of the elution at SO and as S . For example, It is clear that when the share value S , the value of a put option should go to zero. To summaries, equipped with the right boundary conditions. It Is possible using some techniques to solve the BBS equation 1 OFF tort various financial instruments. There are a number tot deterrent solution method one of which I now would like to describe to you.

Transformation To Constant Coefficient Diffusion Equations Physics students may find this subsection interesting. Sometimes it can be useful to transform the basic BBS equation into something a little bit simpler by a change of variables. For example, instead of the function V(S, t), we can introduce a new function according to the following rule V(S, t) = e¤x + ¤¶LLC(X, 6) where or oh=-1 02 - 10, 2 -0 or 10. 000142 Then IS(x, 6) satisfies the basic diffusion equation D U D 21. 1 = 2 . DXL It is a good exercise to check (using your week 8) that the above change of variables equation.

This equation looks much simpler that can be important, for example when simple numerical schemes. Previous 'partial derivative exercises' f mom r indeed gives rise to the standard diffusion than the original BBS equation. Sometimes seeking closed-form lotions, or in some Green's Functions One solution of the BBS equation, which plays a significant role in option pricing, is 1 You can also read about this transformation in the original paper by Black and Schools, a copy of which you can get from me. 7 ? expo 0 for any S'. (Exercise: verify this by substituting back into the BBS equation.) This solution behaves in an unusual way as time t approaches expiration T. You can see that in this limit, the exponent goes to zero everywhere, except at S= S', when the solution explodes. This limit is known as a Doric delta function: lime G(S , t) * 6 (S , S Don not confuse this delta function with the delta of delta hedging!) Think of this as a function that is zero everywhere except at one point, S= S', where it is infinite.

One of the properties of is that its integral is equal to one: +m Another very important property en De TA-donation is where f(S) is an arbitrary function. Thus, the delta-function 'picks up' the value of f at the point, where the delta-function is singular, I. E. At S'= S. How all of this can help us to value financial derivatives? You will see it in a moment. The expression G(S, t) is a solution of the BBS equation for any S'. Because of the linearity of the BBS equation, we can multiply G(S, t) by any constant, and we get another solution.

But then we can also get another solution by adding together expressions of the form G(S, t) but with different values for S'. Putting this together, and taking an integral as Just a way of adding together many solutions, we find that V (S , t)= If(S (S , t)ads ' o m is also a solution of the BBS equation for arbitrary function f(S'). Now if we choose the arbitrary function f(S') to be the payoff function of a given derivative problem, then V(S, t) becomes the value of the option. The function G(S, t) is called the Green's function.

The formula above gives the exact solution for the option value in terms of the arbitrary payoff function. For example, the value of a European call is given by the following integral c(S , t) = f Max(S E (S , t) ads Let us check that as t approaches T the above call option gives the correct payoff. As we mentioned this before, in the limit when t goes to T, the Green's function becomes a delta-function. Therefore, taking the limit we get T , T) = I Max(S E T , S ')ads Max(SST -E , 0). Here we used the property of the delta-function.

Thus, the proposed solution for the call option does satisfy the required boundary condition. Formula For A Call Normally, in financial literature you see a formula for European options written in terms of cumulative normal distribution functions. You may therefore wonder how the exact result given above in terms of the Green's function is related to the ones in the literature. Now I'd like to explain how these two results are related. Let us first focus on a European call. Let us look at the formula for a call c(S , f Max(S E (S , t)ads We integrate from O to infinity. But it is clear that when S'